Attention-Driven Sentiment and the Business Cycle*

Rupal Kamdar
Indiana University, Bloomington

Walker Ray
London School of Economics and CEPR

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Abstract

Using survey data, we show that consumers' economic beliefs are driven by one component, which observationally behaves like "sentiment." Surprisingly, "optimistic" consumers expecting an expansion also predict disinflation, contrasting with professional forecasts. We explain these facts in a New Keynesian model where rationally inattentive consumers face fundamental uncertainty regarding aggregate demand and supply shocks. Optimal information-gathering economizes on information costs but compresses the dimensionality of consumer beliefs. Moreover, because supply-driven recessions are more costly for typical households relying on labor income, more attention is optimally devoted to supply shocks. Inflation is hence perceived as countercyclical; the apparent "sentiment" factor structure of beliefs reflects consumers' optimal focus on aggregate supply shocks. Business cycle dynamics depend crucially on the evolution of aggregate belief misperceptions. Finally, policies which aim to stimulate the economy by raising inflation expectations can have counterproductive consequences.

Keywords: expectations, rational inattention, surveys, business cycles

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1 Introduction

Nearly all economic decisions are based on agents' perceptions about the current economy and expectations about future economic outcomes. Nevertheless, the expectation formation process is still not fully understood. Moreover, surveys of consumer beliefs reveal many puzzling features relative to the predictions of workhorse models of expectations. One surprising correlation in surveys is that consumers who believe unemployment will rise (fall) also expect higher (lower) inflation on average. We document that this "stagflationary" belief correlation is a robust feature over many decades and holds across virtually all demographic consumer groups. Consumer forecasts contrast sharply with the beliefs of professional forecasters (and run counter to the historical comovement between unemployment and inflation in the U.S.).

We show that consumer misperceptions of the typical co-movement between inflation and unemployment are part of a broader phenomenon: the correlation structure of consumer beliefs is almost entirely driven by a single factor. This single factor not only explains consumers' macroeconomic forecasts, but also explains backward-looking beliefs about changes in current economic conditions; beliefs about current and future personal financial conditions; and idiosyncratic views regarding attitudes towards different types of consumption. This factor seemingly behaves like a traditional "sentiment" factor: at any point in time, a given consumer falls on a spectrum between optimism and pessimism. Optimistic consumers forecast typical expansionary outcomes (such as falling unemployment and improving business conditions) as well as improving personal financial conditions. However, if consumers were simply forecasting "demand-driven" booms and busts, otherwise optimistic individuals should predict inflation will rise. Instead, optimistic consumers expect *lower* inflation.

In order to rationalize these puzzling beliefs and better understand their aggregate implications, we next develop a general equilibrium model where agents face frictions when collecting information. We embed a model of rationally inattentive consumers into a two-agent New Keynesian (TANK) framework. Our model features three important frictions: nominal rigidities, financially constrained agents, and costly information. Business cycle fluctuations are driven by aggregate discount rate ("demand") and wage cost-push ("supply") shocks. Because information is costly, agents facing information constraints find it optimal to compress information in the manner which is most informative about their optimal economic actions. We show that households relying on labor income will naturally focus their attention on aggregate supply shocks and away from aggregate demand shocks: supply-driven recessions (where output declines and inflation rises) are particularly harmful for these consumers because optimal labor supply is most sensitive to these shocks. Instead, typical demand-driven recessions (where output declines but inflation falls) feature a natural hedge and so optimal labor supply is less sensitive to these shocks.

Our framework explains why consumers act as if they perceive supply shocks as the dominant driver of the business cycle. It is not that consumers misunderstand the aggregate outcomes of demand shocks; rather, consumers choose to learn more precisely about supply shocks because their consequences are acutely painful. This information acquisition strategy explains why consumer beliefs are explained by a lower dimension factor structure than the data. Consumers receive information about optimal economic actions but then update beliefs about all economic outcomes. From this perspective, the observed degree of "optimism" or "pessimism" of a given consumer is simply a function of this optimal information acquisition.

We show that these information frictions have important implications for the aggregate dynamics of the economy and for policymakers. As in the TANK literature, the existence of hand-to-mouth agents typically implies that aggregate consumption reacts more strongly to shocks than in the representative agent (RANK) benchmark. In our model, as is typical in rational inattention models, the active decisions of information-constrained agents on average under-react relative to the full-information benchmark. However, when hand-to-mouth agents make active decisions about labor supply, consumption can actually over-react in response to shocks, implying *additional amplification* relative to a full-information model. We derive precise analytical characterizations of when information frictions either exacerbate or mitigate these TANK amplification channels. Thus, our model implies that aggregate under- or over-reaction depends on how mistakes due to imperfect information at the household level react with general equilibrium forces. For instance, following a demand-driven expansion, if the optimal full-information response of hand-to-mouth households is to reduce labor supply, then our model implies additional amplification of aggregate output.

Calibrating the model to match important U.S. aggregate business cycle moments and survey data moments, we examine quantitatively how both the aggregate economy and typical beliefs respond to different shocks. Consistent with the intuition described above, the dynamics of inflation beliefs and output beliefs of information-constrained agents are strongly negatively correlated. This is despite the fact that inflation and output are positively correlated in the targeted moments (and thus in the data-generating process of the calibrated model). Additionally, aggregate responses to both demand and supply shocks depend crucially on the ex-ante beliefs of information-constrained households. We compare the dynamics of an economy initially at steady state with a situation where average prior beliefs about a supply-driven recession are two-standard deviations above or below steady state. Output responses to shocks can differ by nearly 50% compared to the model initially at steady state. That is, a shock which boosts output by 1% when average beliefs are at steady state will instead respectively lead to an increase of nearly 1.5% or closer to 0.5% depending on whether information-constrained agents believe a supply-driven expansion or supply-driven recession is likely.

We also consider policies which are aimed at stimulating the economy by manipulating consumer expectations. Typically, such policies seek to induce an increase in consumption through a full-information forward-looking consumption-saving decision. However, we show that policies which increase inflation expectations of information-constrained households can easily backfire: these agents erroneously conclude that inflation will be higher due to an impending supply-driven recession. When the optimal response for these agents is to reduce labor supply, the equilibrium effect is a fall in aggregate output. Quantitatively, we find that a policy which increases the average inflation beliefs of information-constrained agents by 1.0% implies that average output beliefs of these same agents falls by roughly 1.5%. Because information-constrained agents reduce labor supply and consumption, in equilibrium aggregate output also decreases by approximately 0.9%. Thus, our model provides a note of caution for policymakers pursuing polices aimed at manipulating consumer beliefs.

This paper contributes to a number of theoretical and empirical literatures. We study how rational inattention (as in Sims 2003) interacts with financial constraints and nominal rigidities; our contribution is an integrated analysis of the feedback effects of imperfect beliefs and aggregate frictions following shocks. Most closely related to our rational inattention New Keynesian framework are Maćkowiak and Wiederholt (2009), which studies a rationally inattentive firm choosing prices; Maćkowiak and Wiederholt (2015), which studies general equilibrium business cycle dynamics when households and firms are inattentive; and Afrouzi and Yang (2021), which studies the slope of the New Keynesian Phillips curve as a function of inattention frictions in a dynamic model. We additionally focus on the joint dynamics and covariance structure of beliefs and aggregate variables.

Our paper is related to a larger literature which seeks to explain beliefs about business cycle dynamics or economic and financial fluctuations (e.g. see Mankiw and Reis 2002, Carroll 2003, Bordalo et al. 2020, Bordalo et al. 2023, Angeletos and La'O 2013). Outside of the rational inattention literature, most closely related is Bhandari et al. (2024), which studies how agents form beliefs about inflation when subject to model misspecification concerns; time-variation in such concerns imply fluctuations in pessimism which drives biases in consumer beliefs. Our approach is complementary to these papers. In our framework, the centrality of costly information-acquisition has additional implications for heterogeneity across consumer beliefs (absent with representative agents); and explains survey data about *backward-looking* aggregate and personal beliefs (absent in full-information models). We also provide a theoretical justification for empirical findings related to the "causal narratives" of consumers. For instance, households report "greed" and "corporate profits" as drivers of inflation (e.g. see Shiller 1996, Hajdini et al. 2022, Stantcheva 2024); and relative to experts, use supply-side reasoning more (e.g. see Andre et al., 2022, 2023).

Our model is especially well-suited to study policies which either directly or indirectly work by changing household beliefs. This has been a point of discussion among central bankers; ECB Vice-President Vítor Constâncio highlighted the "important role of the central bank in shaping the expectations of the general public". Most papers which study this question do so in the context of forward-looking, full-information rational expectation models (e.g. see McKay et al. 2016). Our model allows us to study a pure "expectation manipulation" policy (also see Coibion et al. 2020a).

Our model also directly extends the theoretical literature on dynamic multivariate rational inattention. We build on the recent theoretical results in Kőszegi and Matějka (2020) (solves a static inattention problem); Maćkowiak et al. (2018) (solves an exogenous scalar inattention problem); Miao et al. (2022) (solves a dynamic multivariate rational inattention problem with individual state variable dynamics). Our contribution extends these analytical results to a model in which aggregate dynamics are endogenous, and depend in equilibrium on the decisions of other information-constrained agents. While the literature on quantitative rational inattention models has developed techniques for solving these models (such as Maćkowiak and Wiederholt 2015), we derive analytical results regarding the factor structure and dynamics of model-implied surveys; implications regarding the sign of belief covariances; and the conditions under which they differ from the data-generating process in general equilibrium.

Empirically, we add to the literature using survey-based expectations to study how agents form beliefs. Coibion et al. (2018a) provide a history of how survey-based measures of beliefs have been used to document deviations from full-information rational expectations (FIRE) such as forecast error predictability or persistent biases.[1] Our empirical contribution to this literature is documenting the robust low-dimension factor structure of consumer beliefs, and how such a factor structure is responsible for the puzzling correlations of consumer inflation and unemployment survey responses.

Finally, we focus on information frictions on the household side; however, similar departures from FIRE have been documented when studying the expectations of firms (e.g. see, Coibion et al. 2018b, Candia et al. 2021). Consistent with our model, Coibion et al. (2020b) show that increases in a given firm's inflation expectations is associated with an increasingly negative outlook regarding business conditions, greater concern about credit accessibility, and increased uncertainty.

2 Empirics

This section presents novel stylized facts about consumer beliefs. We utilize the Michigan Survey of Consumers (MSC) for our main empirical results regarding consumer expectations. The MSC is a long-running consumer survey, which has been conducted monthly since 1978. Typically, the MSC interviews approximately 500 consumers per

[1] Additional empirical studies show important deviations from FIRE: lived experiences affect expectations (e.g. see Malmendier and Nagel 2016); simple heuristics guide expectation formation (Andre et al. 2022). Related research has found that consumers do not understand basic macroeconomic relationships such as the income Fisher equation, the Taylor rule, or the Phillips curve (e.g. see Dräger et al. 2016, Carvalho and Nechio 2014, Jiang et al. 2024).

month. A portion of these respondents are contacted for another survey six months after the initial survey. The MSC asks consumers a range of questions about both aggregate and personal economic conditions that are both forward- and backward-looking.[2]

The majority of MSC questions only allow for categorical responses. For instance, when asking consumers about their beliefs regarding unemployment, the MSC asks, "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?" However, the MSC solicits numerical forecasts when inquiring about consumer beliefs regarding inflation by asking "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?" Section 2.1 analyzes the responses to these two questions; Section 2.2, expands our analysis to a broader set of MSC questions.

As a comparison, we use the Survey of Professional Forecasters (SPF). The SPF is a quarterly survey which began in 1968. Each quarter, approximately 40 professional forecasters are asked to make quantitative forecasts about a range of macroeconomic and financial variables. Forecasters are repeatedly surveyed each quarter, though the composition of respondents changes periodically.

2.1 Inflation and Unemployment Beliefs

We begin by analyzing the relationship between consumer beliefs about unemployment and inflation. Using the MSC, we estimate the following simple regression:

$$\hat{\pi}^{1Y}_{j,t} = \beta^{+}\hat{u}^{+}_{j,t} + \beta^{-}\hat{u}^{-}_{j,t} + \boldsymbol{\gamma}\mathbf{X}_{j,t} + \varepsilon_{j,t}. \tag{1}$$

The dependent variable $\hat{\pi}^{1Y}_{j,t}$ is the one-year ahead inflation forecast of consumer j in month t (numerical response). The indicator variables $\hat{u}^{+}_{j,t}$ and $\hat{u}^{-}_{j,t}$ respectively capture whether the consumer believes unemployment will increase or decrease in the following year (categorical response). Finally, $\mathbf{X}_{j,t}$ denotes a set of time and consumer fixed effects.[3]

Table 1 reports our results. Panel A estimates equation (1) pooled across all

[2] We also conduct all our analyses of consumer beliefs using the Federal Reserve Bank of New York's Survey of Consumer Expectations (SCE) in Appendix B. Overall, the SCE results are qualitatively similar to our baseline MSC analysis.

[3] We winsorize numerical survey responses at the 1% level; results are robust to alternative choices.

Table 1: MSC Inflation and Unemployment Regressions

Panel A:	Baseline		Alternative	
	(1)	(2)	(3)	(4)
Unemployment Up	1.321***	0.383***	0.155***	4.883***
	(0.026)	(0.038)	(0.032)	(0.587)
Unemployment Down	-0.714***	-0.312***	-0.096***	-2.962***
	(0.027)	(0.039)	(0.032)	(0.623)
FEs	N	Y	Y	Y
R-sq	0.125	0.681	0.696	0.723
Obs.	281,034	194,614	152,179	123,183
Panel B:	Income		Investment	
	(1)	(2)	(3)	(4)
Unemployment Up	0.565***	0.258***	0.339	0.285**
	(0.168)	(0.069)	(0.211)	(0.124)
Unemployment Down	-0.545***	-0.265***	-0.578**	-0.108
	(0.192)	(0.069)	(0.258)	(0.107)
FEs	Y	Y	Y	Y
R-sq	0.679	0.697	0.701	0.674
Obs.	16,412	38,388	5,270	10,608
Panel C:	Education		Birth-year	
	(1)	(2)	(3)	(4)
Unemployment Up	0.425***	0.281***	0.436***	0.406***
	(0.084)	(0.068)	(0.064)	(0.101)
Unemployment Down	-0.267***	-0.343***	-0.348***	-0.282***
	(0.094)	(0.069)	(0.069)	(0.097)
FEs	Y	Y	Y	Y
R-sq	0.688	0.677	0.698	0.672
Obs.	48,088	42,959	61,358	25,124

Notes: estimates of equation (1). Columns (1) and (2) of Panel A respectively report our baseline results without and with consumer fixed effects (all regressions include time fixed effects). Columns (3) and (4) replace our dependent variable with 5-year-ahead inflation expectations and 5-year-ahead gas price inflation expectations. Panels B and C repeat our baseline regression, but restrict the sample to different subsets of consumers. Columns (1) and (2) in Panel B restrict to consumers in the bottom and top income quintile; columns (3) and (4) restrict to consumers in the bottom and top quintile of stock holdings. Column (1) of Panel C restricts to consumers without any college education, while column (2) is restricted to consumers with a college degree. Column (3) is restricted to consumers born between 1940 and 1955, while column (4) restricts to consumers born after 1970. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

consumers. Columns (1) and (2) respectively report our baseline regression results without and with consumer fixed effects (all specifications include time fixed effects). In comparison to consumers who expect unemployment will stay the same over the next year (the omitted group), consumers who expect unemployment will rise expect higher inflation, and consumers who expect unemployment will fall expect lower inflation. The difference in inflation forecasts across these consumers is over 2 percentage points on average (and significant at the 1% level).[4] We find consistent results using 5-year-ahead overall inflation expectations (column 3) or 5-year-ahead gas price expectations (column 4) as our dependent variable.

Surprisingly, this belief correlation holds across all consumer demographics. Panels B and C report our estimates of equation (1) restricting the sample to selected subsets of consumers. Our results are robust across the income distribution (Panel A, columns 1 and 2); stock holdings (Panel A, columns 3 and 4); education levels (Panel B, columns 1 and 2); and birth-year cohorts (Panel B, columns 3 and 4).

We compare the correlation in consumer beliefs with those of professional forecasters. Using the SPF, we estimate the following regression:

$$\hat{\pi}^h_{i,t} = \beta \hat{u}^h_{i,t} + \boldsymbol{\gamma} \mathbf{X}_{i,t} + \varepsilon_{i,t}. \tag{2}$$

The dependent variable $\hat{\pi}^h_{i,t}$ is the h-quarter ahead inflation forecast of forecaster i in quarter t (based on the SPF question regarding CPI forecasts, introduced in 1981). The variable $\hat{u}^h_{i,t}$ is based on h-quarter ahead forecast of the unemployment rate. We run this regression separately using forecaster i's unemployment forecast in levels or in differences (relative to the previous quarter).

Table 2: SPF Inflation and Unemployment Regressions

	(1)	(2)	(3)	(4)
Unemployment	-0.126***	-0.152***	-0.283***	-0.143***
	(0.025)	(0.034)	(0.106)	(0.053)
FEs	Y	Y	Y	Y
R-sq	0.895	0.895	0.863	0.863
Obs.	5,399	5,398	5,537	5,536

Notes: estimates of equation (2). Columns (1) and (2) use a one-year forecast horizon; column (1) includes the unemployment forecast in levels and (2) uses differences. Columns (3) and (4) repeat these exercises but use a one-quarter forecast horizon. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Table 2 reports our results. Unlike consumers, professional forecasters who forecast high (or increasing) unemployment tend to believe that inflation will decline. This is true whether we estimate equation (2) using a one-year horizon (columns 1

[4] Including consumer fixed effects, the difference is about 0.7 percentage points. This specification is restrictive as our coefficients of interest are only identified from consumers who are surveyed twice and whose unemployment beliefs change.

and 2) or a one-quarter horizon (columns 3 and 4); or whether we include unemployment forecasts in levels (columns 1 and 3) or in differences (columns 2 and 4).

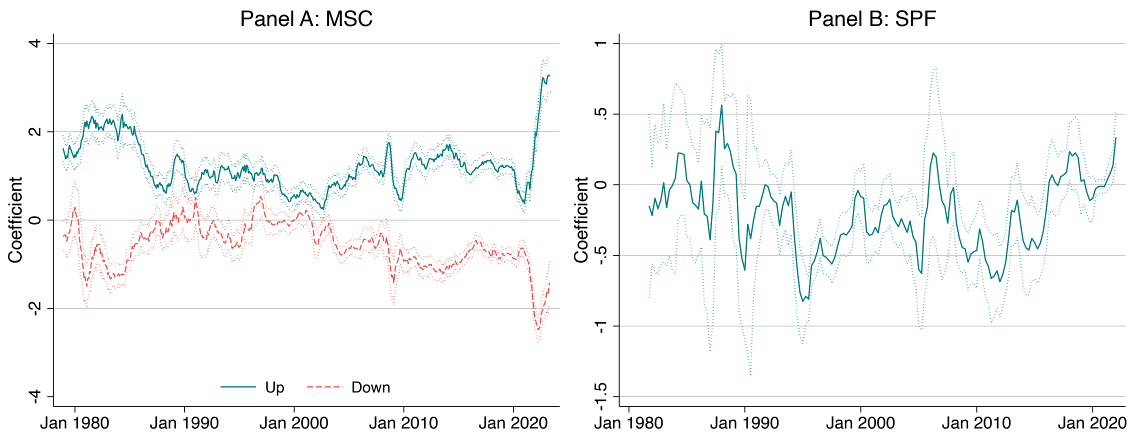

Figure 1: Rolling Inflation/Unemployment Regressions (MSC and SPF)

Notes: 1-year rolling estimates of equations (1) (Panel A) and (2) (Panel B). Each regression includes time fixed effects but no individual fixed effects. For Panel B, we take a 1-year forecast horizon and include SPF forecasts of unemployment in levels. Dotted lines represent 90% confidence intervals.

Figure 1 examines how these estimates vary over time. Panel A estimates (1) using the MSC data (over 12-month rolling windows), while Panel B estimates (2) using the SPF data (over 4-quarter rolling windows). We include time fixed effects but do not include individual fixed effects. The SPF regression uses 1-year-ahead forecasting horizon and includes unemployment forecasts in levels. Panel A shows that across all time periods, consumers who forecast increases in unemployment have higher inflation expectations; the coefficient is positive and significant. Similarly, consumers who believe unemployment will decline have lower inflation forecasts; in all but a handful of periods the coefficient estimate is negative and significant. The magnitudes of the coefficients vary somewhat, but overall the results are remarkably stable. We find a stark contrast with professional forecasters: the coefficient in Panel B is typically negative, while the variation across time is larger than that of consumers.

2.2 Factor Structure of Beliefs

In order to understand the drivers of consumer belief correlations, we dive more deeply into additional responses in the MSC. Beyond inflation and unemployment, the MSC asks consumers for their beliefs regarding a wide range of personal and aggregate economic conditions, as well as attitudes towards consumption. To study the factor structure of consumer beliefs, we conduct a multiple correspondence analysis (MCA) across this much wider range of questions.[5] In our baseline MCA, we include all questions which the MSC has asked continuously since the early 1980s. This includes forward- and backward-looking questions regarding personal financial circumstances; overall economic conditions; and personal attitudes towards different kinds of consumption.

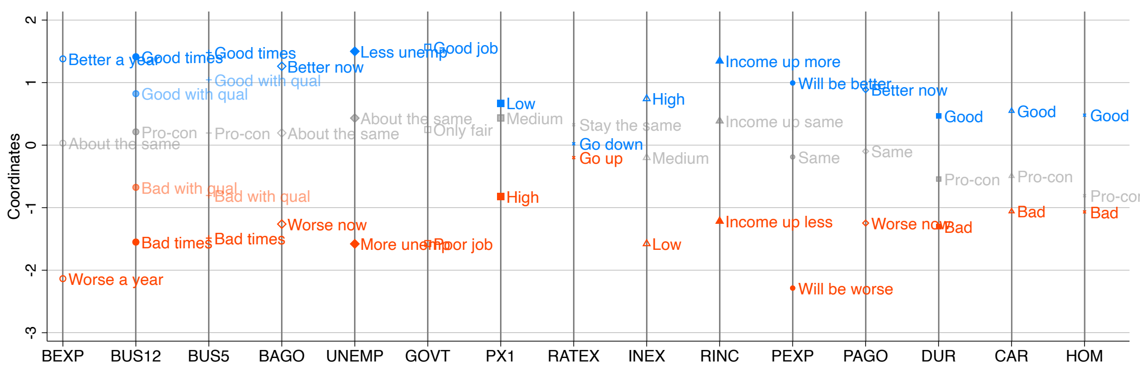

Figure 2: MSC MCA Loadings

Notes: each point represents the estimated loading of the first component for a given categorical response in the baseline MCA. Included questions: business conditions in one year relative to now (BEXP), business conditions over the next year (BUS12), business conditions over the next 5 years (BUS5), business conditions better or worse from a year ago (BAGO), unemployment over the next year (UNEMP), attitudes towards government economic policy (GOVT), inflation over the next year (PX1), interest rates over the next year (RATEX), family income over the next year (INEX), family real income over the next one to two years (RINC), personal financial condition in one year (PEXP), personal financial condition relative to a year ago (PAGO), attitudes towards durable purchases (DUR), attitudes towards auto purchases (CAR), attitudes towards home purchases (HOM). Quantitative questions (PX1 and INEX) are binned into terciles.

First, Figure 2 reports the estimated loadings of the first component in our baseline MCA. Each element of the x-axis is one of the MSC questions included in our MCA analysis; the caption of Figure 2 includes descriptions of all variables included in the MCA. The points on the corresponding vertical line are the estimated loadings for each question's possible responses (labeled in the figure). The estimated loadings paint a very clear picture: responses which are associated with more traditionally "optimistic" outlooks on either personal or aggregate conditions have high and positive loadings (colored in blue); and vice versa, "pessimistic" responses have neg-

[5] MCA is the categorical analogue of principal component analysis (PCA); recall that the majority of questions in the MSC are categorical. The exceptions are inflation forecasts and forecasts of household income, which we bin into terciles when included in the MCA.

ative loadings (colored in red). This is true whether we focus on aggregate forward-looking vs. backward-looking beliefs (e.g. BEXP vs. BAGO); personal or aggregate forecasts (e.g. RINC vs. UNEMP); or across consumption attitudes (e.g. DUR vs. CAR vs. HOM). Our findings regarding inflation and unemployment beliefs are consistent with this pattern as well as our findings in Section 2.1: the estimated loadings for the "less unemployment" and "low inflation" beliefs are positive; while the loadings for "more unemployment" and "high inflation" beliefs are negative. The negative loading on "high inflation" responses is inconsistent with typical aggregate demand-driven business cycle fluctuations, but is apparently consistent with a "sentiment" interpretation where consumers dislike inflation.[6]

Table 3: MSC MCA Summary

Panel A:	Baseline	Additional Prices		Aggregate Only		Personal Only		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dim 1 %	81.9	82.7	82.5	89.3	88.2	66.9	82.4	82.0
Dim 2 %	4.9	4.7	4.9	3.1	3.1	13.6	14.4	16.1
Base Corr.		0.998	0.991	0.928	0.931	0.768	0.632	0.540
Obs.	199,438	125,881	56,166	237,636	139,476	243,752	267,797	278,300
Start Date	1978	1990	2007	1978	1990	1978	1978	1978
Panel B:	Income		Home Value		Investment		Education	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dim 1 %	80.5	80.9	85.4	82.1	82.7	81.1	82.2	81.1
Dim 2 %	5.8	5.0	4.4	4.9	4.2	5.0	5.1	5.2
Base Corr.	0.999	0.999	0.998	0.999	0.999	0.998	0.999	0.999
Obs.	23,024	48,307	10,672	14,327	11,431	14,902	52,749	47,759
Start Date	1979	1979	1990	1990	1990	1990	1978	1978

Notes: Panel A reports MCA results for various questions: (1) baseline; (2) adds 5-year gas questions (introduced in 1990); (3) adds home price questions (introduced in 2007); (4) aggregate questions only; (5) aggregate only, including gas price questions; (6) personal questions only; (7) personal only, excluding consumption questions; (8) personal consumption questions only. Panel B reports MCA results using the baseline set of questions across different respondent subgroups: bottom/top quintiles of income groups (1 and 2); bottom/top quintiles of home value (3 and 4); bottom/top quintiles of stock holdings (5 and 6); and no college/college degree (7 and 8). The baseline correlation is the correlation of fitted first components of a given MCA and the baseline first component.

Even more striking is the fraction of variation the first component explains in survey responses. Table 3 shows that the first component in our baseline MCA

[6]Appendix Figure B3 shows that aggregate time-series fluctuations in this first component (averaged over consumers) is highly correlated with many other measures of "sentiment" in the literature. Additionally, Appendix Figure B4 includes home price expectations (introduced in the MSC in 2007, and only asked to consumers who currently own a home). We find similar results; furthermore, the loadings on home price expectations are such that "high home price inflation" responses are positive while "low home price inflation" responses are negative.

explains over 80% of the variation, while the second component explains less than 5 additional percentage points. This is not driven by our particular choice of questions to include in the MCA. Columns (2) through (8) of Panel A include different sets of questions and report the fraction explained by the first two components. We also report the correlation of the fitted first component in these alternative MCAs with our baseline. Columns (2) and (3) add gas price and home price expectations respectively; columns (4) and (5) restrict the set of questions to only include questions related to aggregate beliefs; while columns (6) through (8) instead restrict the set of questions to only include personal beliefs. The first component always explains the vast majority of variation. Further, the correlation with the baseline fitted component is extremely high. The lowest correlation is for column (8), where we only include the three questions related to attitudes towards consumption (durables, automobiles, and home purchases). However, even in this case the correlation is over 50%.

Panel B shows that the single-dimension factor structure is robust across all demographic groups. Columns (1) and (2) compare the bottom and top quintiles of the income distribution. Columns (3) and (4) compare the bottom and top quintiles of home value. Columns (5) and (6) compare the bottom and top quintiles of stock holdings. Columns (7) and (8) compare consumers with no college education and those with a college degree. Across all groups, the estimated MCAs are highly similar, both in terms of fraction explained and the correlation with our baseline MCA (above 99%).[7]

In comparison, we conduct similar factor analyses of professional forecasters using the SPF. Appendix Table B3 reports a PCA across a wide range of macroeconomic questions in the SPF. There are two major differences from our results using consumer surveys: (i) the first component loading on inflation and unemployment are consistent with demand-driven business cycle fluctuations; and (ii) the first component only explains about 35% of the variation in responses, and the second, third, and fourth components explain over 10 percentage points of variation each.[8]

Taking stock, we find the correlation structure of consumer unemployment and inflation expectations in Section 2.1 is part of a broader phenomenon: consumer

[7]Appendix Figure B5 shows scatter plots of the estimated MCA loadings across different demographic groups. Appendix Figure B6 shows the low-dimension factor structure is stable over time.

[8]Appendix Table B4 also conducts a "pseudo-MCA" using the SPF data by transforming the quantitative responses into quintiles.

beliefs about a wide range of economic and financial conditions are explained by a single component (unlike professionals). Estimated loadings show this component acts like an apparent "sentiment" measure which loads negatively on high inflation beliefs.

3 Model

We now develop a tractable general equilibrium model to rationalize the disconnect between survey-based beliefs and the aggregate fluctuations in economic activity and inflation. The purpose of our model is two-fold. First, we wish to better understand the frictions which drive beliefs. Second, we use our model to explore the aggregate implications of belief frictions.

Our framework builds on standard two-agent New Keynesian (TANK) models (e.g. Bilbiie 2020, Mankiw 2000). Differentiated firms face pricing frictions and produce using labor supplied by households. One set of households has access to financial markets ("savers"), while the other set does not and therefore must consume all income every period ("hand-to-mouth"). Our point of departure is to introduce information frictions: hand-to-mouth households do not have full-information rational expectations. Instead, these households face information frictions as in the rational inattention literature (Sims 2003).[9]

Households: A continuum of households are indexed by $j \in [0, 1]$. For $j \in (\lambda, 1]$, households are "savers" (S). Savers are standard, choosing consumption, labor, and savings in order to maximize lifetime expected utility. We assume these households form expectations with perfect information under rational expectations, denoted by the FIRE operator $\mathbb{E}_t$. The saver households are representative; denoting the representative saver household with superscript 'S', the lifetime discounted expected utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^S, N_t^S; \mathbf{Z}_t\right), \tag{3}$$

[9] Our assumption that the agents subject to financial constraints are also subject to the information constraint is driven by the empirical literature which shows financial constraints reduce cognitive capacity (e.g., Mani et al. 2013, Sergeyev et al. 2023). While financially unconstrained households may also face some degree of information frictions, for tractability, we assume unconstrained households have full information.

and per-period budget constraints are given by

$$C_t^S + Q_t B_t^S = B_{t-1}^S + W_t N_t^S + T_t^S. \tag{4}$$

The S households choose consumption and labor C_t^S, N_t^S and earn the real wage W_t; and choose bond holdings B_t^S with (real) price denoted by Q_t. The final term T_t^S in the budget constraint (4) are transfers from the government and firms. The vector $\mathbf{Z}_t$ collects aggregate preference shifters (described below). In equilibrium, the *representative S household problem* is standard: choose $\left\{C_t^S, N_t^S, B_t^S\right\}_{t=0}^{\infty}$ in order to maximize (3) subject to the sequence of budget constraints (4).

The households $j \in [0, \lambda]$ are "hand-to-mouth" (H), choosing labor and consumption and facing the same per-period utility function $u\left(C_t^{H,j}, N_t^{H,j}; \mathbf{Z}_t\right)$. However, they differ from the representative S households along two dimensions. First, these agents cannot borrow in financial markets and are fully myopic ($\beta^H = 0$), and therefore consume all income every period.[10] Thus, for household $j \in [0, \lambda]$ the household budget constraint is given by

$$C_t^{H,j} = W_t N_t^{H,j} + T_t^H. \tag{5}$$

The real wage W_t is the same for all households, but lump-sum transfers T_t^H will generally differ from S households.

The second difference is that H households face information frictions when forming beliefs. H households cannot observe (current or past) variables perfectly. Instead, H households collect noisy signals $\mathbf{s}_t^j$, but more precise signals are more costly. Expectations of household j are formed with respect to the information set $\{\mathbf{s}_\tau^j\}_{\tau \le t} \equiv \mathcal{I}_t^j$ (the history of signals). We denote the expectation operator of household j by $E_t^j \neq \mathbb{E}_t$ (which differs from FIRE).

Because of information frictions, H households will only observe wages W_t and transfers T_t^H with noise. In order to ensure that the budget constraint (5) binds, we assume that each H household $j \in [0, \lambda]$ consists of workers, shoppers, and a "head of household." At the beginning of the period, the head of household j collects

[10] Under full information, assuming myopia and an inability to borrow is equivalent to assuming an inability to borrow or save. However, information frictions may implicitly introduce a degree of dynamic consideration into the problem of the H households. We return to this point in Section 3.1.

information and forms beliefs about the aggregate economy, which we interpret as "forecasts." The head of household then decides how much labor is supplied by the worker. The shopper receives all labor and transfer income and consumes according to (5). Thus, while the budget constraint binds with equality in each period, no new information is revealed to the head of household at the end of each period. This implies that labor supply $N_t^{H,j}$ is the active choice of the H households, while consumption $C_t^{H,j}$ acts as a residual. With this assumption, the H household payoff function can be written

$$E_t^j \, U\left(N_t^{H,j}; \mathbf{X}_t\right) - \mu I\left(\mathbf{X}_t; \mathcal{I}_t^j \,\middle|\, \mathcal{I}_{t-1}^j\right). \tag{6}$$

Concentrated utility is defined by $U\left(N_t^{H,j}; \mathbf{X}_t\right) = u\left(W_t N_t^{H,j} + T_t^H, N_t^{H,j}; \mathbf{X}_t\right)$, and depends on the labor choice $N_t^{H,j}$ as well as $\mathbf{X}_t$, the set of all aggregate variables relevant for the H household decisions. The vector $\mathbf{X}_t$ will contain the preference shifters $\mathbf{Z}_t$ and any other state variables or shocks which affect the real wage W_t and transfers T_t^H. Both the set and distribution of variables $\mathbf{X}_t$ is endogenous but taken as given by households. The final term captures information costs. Information costs depend on $I\left(\mathbf{X}_t; \mathcal{I}_t^j \,\middle|\, \mathcal{I}_{t-1}^j\right)$, the conditional Shannon mutual information between the variables $\mathbf{X}_t$ and the signals in the current information set $\mathcal{I}_t^j$, given the previous history of signals in $\mathcal{I}_{t-1}^j$. We assume information costs are a linear function of conditional Shannon mutual information; the coefficient μ therefore captures how costly is an additional "unit" of information. In equilibrium, the *hand-to-mouth household j problem* is to maximize (6) by choosing both a distribution of signals $\mathbf{s}_t^j$ and labor supply $N_t^{H,j}$, taking the information set $\mathcal{I}_{t-1}^j$ as given.

Firms: Differentiated intermediate goods are produced by a continuum of monopolistically competitive firms $i \in [0,1]$ producing output $Y_t(i)$. The final consumption basket is produced by a representative firm in a perfectly competitive retail sector, which combines differentiated products using the usual constant elasticity of substitution. This implies that the consumption basket C_t^j for household j is given by $C_t^j = \left[\int_0^1 C_t^j(i)^{(\epsilon-1)/\epsilon} \, \mathrm{d}i\right]^{\epsilon/(\epsilon-1)}$. Demand for good i from household j is therefore $C_t^j(i) = \left(P_t(i)/P_t\right)^{-\epsilon} C_t^j$, where $P_t(i)$ is the price chosen by firm i and $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} \, \mathrm{d}i\right]^{1/(1-\epsilon)}$ is the price index. Aggregate demand for good i is therefore given by $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$, where C_t is aggregate consumption.

Intermediate firms produce using a linear technology in labor $Y_t(i) = N_t(i)$, which they hire at the real wage W_t. Firms choose prices in order to maximize discounted expected profits, but face Calvo pricing frictions: a firm cannot update its price each period with probability θ (iid across time and firms). We assume intermediate firms are owned by the S households. When updating prices $P_t(i)$, lifetime expected discounted profits are given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q^S_{t,t+k} D_{t+k|t}(i), \tag{7}$$

where real profits of firm i are $D_{t+k|t}(i) = (1+\tau^S)\left(P_t(i)/P_{t+k}\right) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) - T^F_{t+k}$ if the firm is unable to update its price from $P_t(i)$ at time $t+k$. Thus, profits are discounted by θ^k, the probability of being unable to change prices from $P_t(i)$ at time $t+k$. The term $Q^S_{t,t+k}$ is the real SDF of the S households (where expectations are taken under FIRE, consistent with S households). Profits include a production subsidy τ^S, financed by lump-sum taxes T^F_t. In equilibrium, the *firm i problem* when updating prices (with probability $1-\theta$) is to choose $P_t(i)$ in order to maximize (7), subject to the production technology and the sequence of CES demand constraints.

Government: The fiscal authority sets an optimal production subsidy $\tau^S = 1/(\epsilon - 1)$, implying markups are zero in steady state. This subsidy is self-financed with firm lump-sum taxes: $T^F_t = \int_0^1 \tau^S \left(P_t(i)/P_t\right) Y_t(i)\, \mathrm{d}i$. The fiscal authority also taxes the profits of S households at a rate τ^D and redistributes to the H households. Aggregate profits $D_t = \int_0^1 D_t(i)\, \mathrm{d}i$ are received each period by the S households, so a given S household pays a tax $\tau^D D_t/(1-\lambda)$, while a given H household receives $\tau^D D_t/\lambda$. The central bank chooses the nominal interest rate $i_t \equiv -\log Q^{(nom)}_t$, where $Q^{(nom)}_t$ is the price of a nominal one period bond.

Aggregate Shocks: Per-period utility is separable in consumption and labor and depends on a vector of aggregate shocks $\mathbf{Z}_t \equiv (\Psi_t, \Gamma_t)$:

$$u(C^j_t, N^j_t; \mathbf{Z}_t) = \Psi_t \left[\frac{\left(C^j_t\right)^{1-\varsigma} - 1}{1-\varsigma} - \Gamma_t \frac{\left(N^j_t\right)^{1+\varphi}}{1+\varphi} \right]. \tag{8}$$

Thus, Ψ_t is an aggregate discount factor shock and Γ_t is an aggregate disutility of labor shock which affect all households (but note that Ψ_t plays no direct role in H decisions).

We assume this set of aggregate shocks in order to parsimoniously map our model to the empirical results in Section 2. Our choices are driven by three main considerations. First, we want sets of shocks which may have different qualitative effects on output and inflation. As we will show, in equilibrium Ψ_t will act as an "aggregate demand" shock, while Γ_t will act as a wage cost-push "aggregate supply" shock. Second, we assume that the wage cost-push shock takes the form of a labor disutility shock so that the H household decisions are directly affected by Γ_t. Third, we abstract from more standard technology shocks so that we can work directly with output rather than output gaps.

Aggregation and Linearization: Aggregating across firms and S households is standard. However, while the H households are identical in terms of preferences, expectations may differ across H households, and so consumption and labor choices may differ as well. Define the average consumption and labor supply of the H households as $C_t^H \equiv \frac{1}{\lambda}\int_0^\lambda C_t^{H,j}\,\mathrm{d}j$ and $N_t^H \equiv \frac{1}{\lambda}\int_0^\lambda N_t^{H,j}\,\mathrm{d}j$. Aggregate consumption and labor supply are thus $C_t = \lambda C_t^H + (1-\lambda)C_t^S$ and $N_t = \lambda N_t^H + (1-\lambda)N_t^S$.

We approximate the model around the zero-inflation steady state. Where applicable, lower case variables denote log-deviations from steady state values: $X_t = \bar{X}e^{x_t}$. For profits which are zero in steady state, define $d_t = D_t/\bar{Y}$. For now, we take as given the average labor choice of H households, which allows us to defer solving the H information problem. Given N_t^H, dynamics in our model mimic standard TANK models (see Appendix C for additional derivations).

Because the optimal production subsidy ensures profits are zero in steady state, consumption and labor supply decisions of H and S households will also be equal in steady state. Thus, the log-linearized aggregate consumption and labor supply equations are simply $c_t = \lambda c_t^H + (1-\lambda)c_t^S$ and $n_t = \lambda n_t^H + (1-\lambda)n_t^S$. Market clearing in goods markets and production also implies that $y_t = c_t = n_t$ (since price dispersion has no first-order effects on aggregate output). Aggregate profits are given by $d_t = -w_t$. The representative S intratemporal and intertemporal optimality conditions take the usual log-linearized form

$$w_t = \gamma_t + \varsigma c_t^S + \varphi n_t^S, \;\; \mathbb{E}_t \Delta c_{t+1}^S = \varsigma^{-1} \left(i_t - \mathbb{E}_t \pi_{t+1} - v_t \right), \tag{9}$$

where $v_t = \mathbb{E}_t \Delta \psi_{t+1}$ and the policy rate i_t is measured as deviations from the long-run rate $i^* \equiv -\log \beta$. The aggregate discount factor and wage cost-push shocks follow independent AR(1) processes $v_t = \rho_v v_{t-1} + \varepsilon_{v,t}$ and $\gamma_t = \rho_\gamma \gamma_{t-1} + \varepsilon_{\gamma,t}$, where $\varepsilon_{v,t} \sim \mathcal{N}(0, \sigma_v^2)$ and $\varepsilon_{\gamma,t} \sim \mathcal{N}\left(0, \sigma_\gamma^2\right)$ are iid Gaussian innovations.

Log-linearized firm optimality conditions imply a New Keynesian Phillips curve

$$\pi_t = \kappa_w w_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{10}$$

where $\kappa_w \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ is the slope of the Phillips curve with respect to marginal cost (which in our model is given by the real wage).

From the H budget constraint, we have that $c_t^H = n_t^H + \left(1 - \tau^D/\lambda\right) w_t$ (where the term $-\tau^D w_t/\lambda$ captures any fiscal redistribution and follows because aggregate profits are inversely related to the wage). Under full information, the H and S intratemporal optimality conditions are the same; H optimal decisions are given by

$$(\varsigma + \varphi) n_t^{H*} = \chi_n w_t - \gamma_t, \;\; (\varsigma + \varphi) c_t^{H*} = \chi_c w_t - \gamma_t, \tag{11}$$

$$\text{where} \quad \chi_n \equiv 1 - \varsigma\left(1 - \tau^D/\lambda\right), \;\; \chi_c \equiv 1 + \varphi\left(1 - \tau^D/\lambda\right). \tag{12}$$

Our model will feature similar departures from standard RANK models as in Bilbiie (2020). Indeed, under full information our model only differs in terms of the shocks we consider; in particular, the role played by the parameter χ_c is identical in terms of the dynamics of output to a demand shock. However, because of information frictions, in general household j will choose $n_t^{H,j} \neq n_t^{H,*}$; moreover, average labor supply in equilibrium will also differ from the full-information case $n_t^H \neq n_t^{H,*}$. Thus, $w_t \neq \gamma_t + (\varsigma + \varphi) y_t$ (as would be the case under full information). Instead, combining market clearing conditions with S intratemporal optimality conditions and the H budget constraint, we have

$$w_t = \frac{(1-\lambda)\gamma_t + (\varsigma + \varphi)(y_t - \lambda n_t^H)}{1 - \lambda \chi_n} \equiv \omega_\gamma \gamma_t + \omega_y y_t + \omega_n n_t^H. \tag{13}$$

Thus, the real wage (and therefore firm marginal costs) will be affected by the information frictions faced by H households (since equilibrium wages depend directly on the labor supply decisions of all households). Combining equilibrium H consumption with the S intertemporal choices, aggregate output evolves according to

$$\mathbb{E}_t \Delta y_{t+1} = \frac{(1-\lambda)\varsigma^{-1}}{1-\lambda\zeta_y}\left(i_t - \mathbb{E}_t \pi_{t+1} - v_t\right) + \frac{\lambda\zeta_\gamma}{1-\lambda\zeta_y}\mathbb{E}_t \Delta\gamma_{t+1} + \frac{\lambda\zeta_n}{1-\lambda\zeta_y}\mathbb{E}_t \Delta n^H_{t+1}, \quad (14)$$

and the Phillips curve can be written

$$\pi_t = \kappa_w \left[\omega_\gamma \gamma_t + \omega_y y_t + \omega_n n_t^H\right] + \beta \mathbb{E}_t \pi_{t+1}. \quad (15)$$

Hence, aggregate dynamics will depart from RANK for similar reasons as in TANK models. For instance, the output elasticity with respect to the interest rate is no longer given by the intertemporal elasticity of substitution; and wage cost-push shocks appear directly in (14). But aggregate dynamics will also depend on the dynamics of the labor supply decisions of information-constrained households. To understand the differences from full-information TANK models, we next derive how information-constrained agents learn about the economy.

3.1 Belief Factor Structure: General Results

Before solving for the equilibrium dynamics of our specific model, this section studies the belief structure of our model's inattentive agents. Taking as given the aggregate dynamics of the model, we characterize how household beliefs are formed under very general conditions. This allows us to illuminate which results regarding beliefs will hold under alternative models (such as different shocks or different choice sets of households); and vice versa, which elements of our model are necessary for matching the facts we document in Section 2.

Suppose that the equilibrium aggregate dynamics of the model can be written

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbb{E}_t \mathbf{y}_{t+1} \end{bmatrix} = \tilde{\mathbf{A}} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{y}_t \end{bmatrix} + \tilde{\mathbf{C}} \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \quad (16)$$

The vector $\mathbf{x}_t$ collects all predetermined (state) variables, $\mathbf{y}_t$ collects all nonpredetermined (jump) variables, and $\boldsymbol{\varepsilon}_t$ collects all innovation (shock) variables. Gaussian shocks ensure that the information problem is tractable (but assuming independence is without loss of generality). We assume the usual Blanchard and Kahn (1980) determinacy conditions hold, so there exists a unique linear rational expectations equilibrium.

The dynamics matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{C}}$ are endogenous, as are the set of state variables; in equilibrium, these objects will depend on how inattentive households collect information (we return to this point below). Nevertheless, all agents take these dynamics as given. This includes the information-constrained households: while they do not observe variables perfectly, they fully understand the dynamics of the model conditional on the realization of the state and shocks.

We maintain the following assumptions: an inattentive agent j is (i) "hand-to-mouth" (i.e., there are no idiosyncratic state variables such as savings); and (ii) myopic (i.e., discount factor $\beta^j = 0$). We solve the information problem using an arbitrary per-period utility function which may depend directly on state variables $\mathbf{x}_t, \mathbf{x}_{t-1}$, jump variables $\mathbf{y}_t$, and realizations of the shock $\boldsymbol{\varepsilon}_t$. We further allow for a more generic set of actions $\mathbf{a}_t^j$ (and where the set of actions has already concentrated out any constraints). The following Proposition characterizes the dynamics of beliefs and actions.

Proposition 1 (Optimal Information, General Dynamics)**.** *Suppose equilibrium dynamics are described by equations (16). Then the information costs of a myopic agent j are given by $\mu I\left(\mathbf{X}_t; \mathcal{I}_t^j \middle| \mathcal{I}_{t-1}^j\right)$ where the vector $\mathbf{X}_t$ satisfies*

$$\begin{bmatrix} \mathbf{x}_{t-1} \\ \boldsymbol{\varepsilon}_t \end{bmatrix} \equiv \mathbf{X}_t = \begin{bmatrix} \mathbf{A}_x & \mathbf{C}_x \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{X}_{t-1} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \boldsymbol{\varepsilon}_t \equiv \mathbf{A}\mathbf{X}_{t-1} + \mathbf{C}\boldsymbol{\varepsilon}_t, \tag{17}$$

and matrices $\mathbf{A}_x, \mathbf{C}_x$ are defined in (A1). The quadratic utility approximation

$$U\left(\mathbf{x}_t, \mathbf{x}_{t-1}, \mathbf{y}_t, \boldsymbol{\varepsilon}_t; \mathbf{a}_t^j\right) \approx -\left(\mathbf{a}_t^j\right)^\top \mathbf{B}_{aa}\mathbf{a}_t^j + \mathbf{X}_t^\top \mathbf{B}_{ax}\mathbf{a}_t^j \tag{18}$$

implies the optimal signal structure is a (time-invariant) linear Gaussian process:

$$\mathbf{s}_t^j = \mathbf{H}_x\mathbf{x}_{t-1} + \mathbf{H}_\epsilon\boldsymbol{\varepsilon}_t + \boldsymbol{\eta}_t^j \equiv \mathbf{H}\mathbf{X}_t + \boldsymbol{\eta}_t^j, \quad \boldsymbol{\eta}_t^j \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_\eta\right), \tag{19}$$

with associated prior and posterior covariances and Kalman gain matrix, respectively *denoted by* $\mathbf{\Sigma}_{1|0} \equiv Var_t\left[\mathbf{X}_t | \mathcal{I}_{t-1}^j\right]$ *and* $\mathbf{\Sigma}_{1|1} \equiv Var_t\left[\mathbf{X}_t | \mathcal{I}_t^j\right]$*, and* $\mathbf{K}$*, jointly solving the Kalman filter equations* (A2)*. Posterior means evolve according to*

$$\hat{\mathbf{X}}_t^j \equiv E_t\left[\mathbf{X}_t | \mathcal{I}_t^j\right] = \mathbf{KHX}_t + (\mathbf{I} - \mathbf{KH})\,\mathbf{A}\hat{\mathbf{X}}_{t-1}^j + \mathbf{K}\boldsymbol{\eta}_t^j, \tag{20}$$

and prior means are given by $\tilde{\mathbf{X}}_t^j \equiv E_t\left[\mathbf{X}_t | \mathcal{I}_{t-1}^j\right] = \mathbf{A}\hat{\mathbf{X}}_{t-1}^j$*. More generally, k-step ahead forecasts are given by* $\hat{\mathbf{X}}_{t+k|t}^j \equiv E_t\left[\mathbf{X}_{t+k} | \mathcal{I}_t^j\right] = \mathbf{A}^k\hat{\mathbf{X}}_t^j$*.*

Optimal actions are given by $\mathbf{a}_t^j = \frac{1}{2}\mathbf{B}_{aa}^{-1}\mathbf{B}_{ax}\hat{\mathbf{X}}_t^j$*. The optimal signal coefficient and covariance matrix choices depend on the eigendecomposition of the loss matrix*

$$\mathbf{\Omega} \equiv \frac{1}{4}\mathbf{B}_{ax}\mathbf{B}_{aa}^{-1}\mathbf{B}_{ax}^\top, \quad \mathbf{\Sigma}_{1|0}^{1/2}\mathbf{\Omega}\mathbf{\Sigma}_{1|0}^{1/2} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\top. \tag{21}$$

Let $\mathbf{\Lambda}_1$ *be the eigenvalues satisfying* $\Lambda_i > \frac{1}{2}\mu$*, and* $\mathbf{U}_1$ *the associated eigenvectors. One choice of optimal* $\mathbf{\Sigma}_\eta$ *is a diagonal matrix with elements given by* $\sigma_{\eta,i}^2 = (2\Lambda_i/\mu - 1)^{-1}$*. The corresponding signal coefficient matrix is then given by* $\mathbf{H} = \mathbf{U}_1^\top\mathbf{\Sigma}_{1|0}^{-1/2}$*.*

All proofs are in Appendix A. The proof builds heavily on existing results in the rational inattention literature. Because the inattentive agents in our model have no idiosyncratic state variables and are myopic, the inattentive problem is very similar to a repeated static problem as in Kőszegi and Matějka (2020) or the dynamic generalization as in Miao et al. (2022). The difficulty of our setting is that the preferences and dynamics of our model contain both forward- and backward-looking variables (e.g., inflation and output are nonpredetermined, while the aggregate discount and wage cost-push factors are predetermined). However, once we have correctly specified the state space of the problem, Proposition 1 follows naturally.

The intuition behind the structure of how inattentive agents obtain information is the usual "water-filling" approach. Obtaining information is costly, but doing so helps agents make better economic choices. Instead of obtaining independent signals about each fundamental, inattentive consumers economize on information costs and reduce the dimensionality of the problem by learning about combinations of fundamentals in the manner which is most useful for taking optimal actions. The logic of the "water-filling" solution to the information problem implies that the factor structure of posterior beliefs may be lower than that of the data-generating process. An immediate corollary is that this will always hold for the H households in our model.

Corollary 1.1 (Hand-to-Mouth Optimal Signal)**.** *The loss matrix* $\mathbf{\Omega}$ *from* (21) *of a hand-to-mouth household* j *described in* (6) *has one non-zero eigenvalue* Λ_1*. If* $\Lambda_1 > \frac{1}{2}\mu$*, then the optimal signal can be written* $s_t^j = n_t^{H,*} + \eta_t^j$*, where the variance of the signal noise is* $\sigma_\eta^2 = (2\Lambda_1/\mu - 1)^{-1}$*. The prior and posterior mean jointly evolve according to*

$$\hat{n}_t^{H,j} = K(n_t^{H,*} + \eta_t^j) + (1-K)\tilde{n}_t^{H,*,j}, \quad K \equiv \frac{1}{1+\sigma_\eta^2}. \tag{22}$$

If instead $\Lambda_1 < \frac{1}{2}\mu$*, the agent receives no information and* $\hat{n}_t^{H,j} = \tilde{n}_t^{H,*,j} = 0$*.*

Recall that H households only make one active decision: how much labor to supply. Thus, when deciding to collect more information, it will always be optimal to learn more precisely about what this optimal choice is. Any other information which does not assist in this decision is therefore extraneous, and due to the cost of acquiring additional information will be ignored in equilibrium. From Proposition 1, the optimal signal weights on the unobserved state will be (proportional to) the eigenvector of $\mathbf{\Omega}$ associated with the only non-zero eigenvalue.

The assumption that H households are myopic ($\beta^j = 0$) is important for Corollary 1.1. This may seem surprising: under full information, optimal future actions $n_{t+k}^{H,*}$ are independent of previous decisions taken by the household. However, information is carried into the future and may be useful not only for the optimal action today, but also future actions. When the dynamics of the optimal action are sufficiently rich, non-myopic agents ($\beta^j > 0$) will take these dynamic considerations into account (see Maćkowiak et al. 2018). We maintain the assumption of myopic hand-to-mouth agents not only for tractability, but also because this is an empirically relevant assumption for traditional "Keynesian" hand-to-mouth agents (see Aguiar et al. 2024).

In order to map beliefs in our model to the empirical results, we formally represent "surveys" as functions of the variation in posterior beliefs. Denote the long-run covariance of the data-generating process and posterior beliefs regarding $\mathbf{X}_t$ as $\mathbf{\Sigma}_X \equiv \mathbb{V}ar\left[\mathbf{X}_t\right]$ and $\mathbf{\Sigma}_{\hat{X}} \equiv \mathbb{V}ar\left[\hat{\mathbf{X}}_t^j\right]$, respectively. Similarly, define the conditional covariances as $\check{\mathbf{\Sigma}}_X \equiv \mathbb{V}ar\left[\mathbf{X}_t \middle| \mathbf{X}_{t-1}\right]$ and $\check{\mathbf{\Sigma}}_{\hat{X}} \equiv \mathbb{V}ar\left[\hat{\mathbf{X}}_t^j \middle| \mathbf{X}_{t-1}, \hat{\mathbf{X}}_{t-1}^j\right]$. The long-run and conditional covariances of jump variables $\mathbf{y}_t$ are defined analogously, denoted by $\mathbf{\Sigma}_y$ and $\check{\mathbf{\Sigma}}_y$. In all cases, these covariances are computed with respect to the physical dynamics (17) and (20). Given the timing assumption of information collection, we interpret $\hat{\mathbf{X}}_t^j$ as the forecast of household j (though using the results from Proposition 1 we can extend these results to k-step ahead forecasts $\hat{\mathbf{X}}_{t+k|t}^j$).

It is immediately clear that when faced with information-processing frictions, the distribution of survey-based beliefs will not be equivalent to the physical distribution from the data-generating process. More surprisingly, these differences can persist for even arbitrarily small information costs, as we show in the next Proposition.

Proposition 2 (Survey Belief Distribution, General Dynamics)**.** *Whenever information costs $\mu > 0$, long-run covariances of posterior beliefs differ from the data-generating process: $\mathbf{\Sigma}_{\hat{X}} \neq \mathbf{\Sigma}_X$ and $\mathbf{\Sigma}_y \neq \mathbf{\Sigma}_{\hat{y}}$. Moreover, if $\mathbf{\Omega}$ is not full rank, these distributions differ even in the limit as information costs disappear: $\lim_{\mu\to 0} \mathbf{\Sigma}_{\hat{X}} \neq \mathbf{\Sigma}_X$ and $\lim_{\mu\to 0} \mathbf{\Sigma}_{\hat{y}} \neq \mathbf{\Sigma}_y$. For any $\mu > 0$, the rank of posterior belief conditional covariances $\operatorname{rank} \check{\mathbf{\Sigma}}_{\hat{X}}, \operatorname{rank} \check{\mathbf{\Sigma}}_{\hat{y}}$ are bounded above by $\operatorname{rank} \mathbf{\Omega}$.*

Our results thus far are consistent with our empirical findings: household beliefs are well-described by a single factor, and the covariance of survey-based beliefs regarding aggregate variables will generally differ from that of the underlying data. However, the conditions under which the correlation of output and inflation in the data and in surveys have different signs will depend on the specifics of the model. Nevertheless, the following Corollary derives two general results in the case when signals are one-dimensional and the dynamics of the model are iid.

Corollary 2.1 (Hand-to-Mouth Posterior Belief Dynamics)**.** *Suppose that the dynamics matrix $\mathbf{A}_x = \mathbf{0}$ and the loss matrix $\mathbf{\Omega}$ has only one eigenvalue which satisfies $\Lambda_1 > \frac{1}{2}\mu$, with associated eigenvector $\mathbf{u}_1$ and signal coefficient vector $\mathbf{h} = \mathbf{u}_1^\top \mathbf{\Sigma}_{1|0}^{-1/2}$.*

(i) If $\mathbf{h} \propto \mathbf{e}_k^\top$ (the k-dimension standard basis vector) so that $\mathbf{h}\mathbf{X}_t \propto x_{t,k}$, then for any two jump variables $y_{1,t}, y_{2,t} \in \mathbf{y}_t$, $\operatorname{sign} \mathbb{C}ov\left(\hat{y}_{t,1}^j, \hat{y}_{t,2}^j\right) = \operatorname{sign} \frac{\partial y_{t,1}}{\partial x_{t,k}} \cdot \frac{\partial y_{t,2}}{\partial x_{t,k}}$.

(ii) If $\mathbf{h}\mathbf{X}_t \propto y_{t,1}$ for some jump variable $y_{t,1} \in \mathbf{y}_t$, then for any other jump variable $y_{t,2} \in \mathbf{y}_t$, $\operatorname{sign} \mathbb{C}ov\left(\hat{y}_{t,1}^j, \hat{y}_{t,2}^j\right) = \operatorname{sign} \mathbb{C}ov\left(y_{t,1}, y_{t,2}\right)$.

The iid assumption in Corollary 2.1 simplifies the proof, but the intuition behind these results holds under more complicated dynamics (and also extends to k-step ahead forecasts $\hat{\mathbf{X}}_{t+k|t}^j$). Result (i) says that if information-constrained agents learn only about one single state variable $x_{t,k}$, then the covariance between any jump variables in survey beliefs will have the same sign as the conditional response of these variables to $x_{t,k}$. Result (ii) on the other hand says that if agents are effectively learning about only one single jump variable $y_{t,1}$, then the covariance between this and any other jump variables in survey beliefs will have the same sign as the (actual) unconditional covariance of these variables. Intuitively, in either case agents are only learning about one single aggregate variable, and so conclusions about any other aggregate variable can only be drawn based on how variables endogenously covary within the model. In (i), this implies that beliefs about other aggregate variables are based on the (actual) conditional response to $x_{t,k}$. In (ii), this implies that beliefs about other variables are based on (actual) unconditional covariances.

We are now in a position to apply our results to the findings of Section 2. Propositions 1 and 2 show that dimension-reduction is a natural way for agents to economize on information costs, and so posterior beliefs will naturally feature a smaller factor structure than the data-generating process. Corollary 1.1 applies these findings to information-constrained agents in our model, who always reduce the dimensionality of the information problem to at most one dimension, consistent with a single "sentiment" factor. The conditions under which output and inflation beliefs negatively covary will depend on the specifics of the model.[11] Corollary 2.1 shows that even when the unconditional covariance between output and inflation in the data-generating process is positive, (i) gives us possible conditions under which surveys will show negative correlation of output and inflation beliefs. In particular, when inattentive agents find it optimal to pay attention to shocks which cause inflation and output to negatively covary, survey beliefs will feature this same negative covariance. On the other hand, (ii) shows that if agents find it optimal to effectively pay attention only to output (or inflation), then the covariance of output and inflation beliefs will necessarily match the data-generating process.

State Space Representation: Before we can determine the equilibrium dynamics of our model, note that in solving the information problem, we implicitly assumed the model can be represented by a (finite) set of state variables. This assumption is

[11]Since we do not explicitly model unemployment, we proxy these beliefs as inversely related to beliefs about output (as output moves one-for-one with aggregate labor supply in the model).

not innocuous, even in our simple case with hand-to-mouth agents solving (6). To see why, define the average of H households' prior beliefs of optimal labor supply by

$$m_t \equiv \frac{1}{\lambda} \int_0^\lambda \tilde{n}_t^{H,*,j} \, \mathrm{d}j \, . \tag{23}$$

Corollary 1.1 implies aggregate H labor supply is $n_t^H = \frac{K}{\varsigma+\varphi} (\chi_n w_t - \gamma_t) + (1-K)m_t$. But then from (13), in equilibrium the real wage is a function of the average labor choice n_t^H and thus implicitly depends on average priors m_t. Hence, average priors are an endogenous state variable, and so in general the optimal signal will place non-zero weight on m_t. H households form prior and posterior beliefs $\tilde{m}_t^j, \hat{m}_t^j$ which will implicitly affect their labor supply decision $n_t^{H,j}$; in turn, average priors about these objects are themselves state variables, and so on. Even in our simple case of myopic hand-to-mouth agents, information frictions lead to an "infinite regress" problem.[12] However, our goal is to develop a tractable model, and fortunately there are special cases which allow us to sidestep this issue. Section 4 simplifies to the case of iid shocks. Section 5 allows for more complicated dynamics under certain parametric restrictions.

4 Analytical Results

In this section, we focus on the case of iid shocks. When the exogenous structural factors are iid, the Kalman filtering problem of inattentive consumers is simple, as priors are always at steady state values. We therefore avoid the dynamic complexity of the evolution of aggregate prior beliefs and are able to derive clear analytical results.

With iid dynamics ($\rho_v = \rho_\gamma = 0$), the state space is simply given by $\mathbf{X}_t \equiv \begin{bmatrix} v_t & \gamma_t \end{bmatrix}^\top$, and Proposition 1 implies that H household prior beliefs will always equal steady state values. Then Corollary 1.1 implies that the average labor supply decision of H households is simply $n_t^H = K n_t^{H,*}$. Combining this with equations (11) and (13), the equilibrium wage and H consumption are a function of wage cost-push shocks and output $w_t = \tilde{\omega}_\gamma \gamma_t + \tilde{\omega}_y y_t$, and $c_t^H = \tilde{\zeta}_\gamma \gamma_t + \tilde{\zeta}_y y_t$, where the parameters $\tilde{\omega}_\gamma, \tilde{\omega}_y, \tilde{\zeta}_\gamma, \tilde{\zeta}_y$ are defined in equations (C9)-(C12).

Finally, we assume that the central bank follows a simple Taylor rule $i_t = \phi_\pi \pi_t$,

[12] Dynamics follow a vector $AR(\infty)$ process which can be approximated by an $ARMA(p,q)$ process (see e.g. Maćkowiak and Wiederholt 2015).

where i_t represents the deviations from the steady state interest rate $i^* = -\log\beta$, consistent with the zero inflation steady state. Then assuming ϕ_π is large enough, determinacy conditions are met; the assumption of white noise shocks implies that S expectations about future aggregate variables are always at steady state: $\mathbb{E}_t y_{t+1} = \mathbb{E}_t \pi_{t+1} = 0$. Equations (14) and (15) become

$$y_t = \frac{(1-\lambda)\varsigma^{-1}}{1-\lambda\tilde{\zeta}_y}(v_t - \phi_\pi \pi_t) + \frac{\lambda\tilde{\zeta}_\gamma}{1-\lambda\tilde{\zeta}_y}\gamma_t, , \quad \pi_t = \kappa_w \tilde{\omega}_\gamma \gamma_t + \kappa_w \tilde{\omega}_y y_t. \tag{24}$$

Inverting this system characterizes the equilibrium response of output and inflation to discount factor and wage cost-push shocks:

$$y_t = C_{y,v} v_t + C_{y,\gamma}\gamma_t, \quad \pi_t = C_{\pi,v} v_t + C_{\pi,\gamma}\gamma_t, \tag{25}$$

where the expressions for the coefficients are given by equations (C5)-(C8).

4.1 Beliefs

The following Proposition shows the conditions under which posterior beliefs feature negative correlation between output and inflation, while (unconditional) correlations are positive. We show that beliefs depend crucially on $\chi_n = 1 - \varsigma\left(1 - \tau^D/\lambda\right)$, which governs how the optimal labor supply decision of H households varies as a function of the real wage (11). Note that from Proposition 1, with iid shocks the k-step ahead forecasts of inattentive agents will always return to steady state. However, recall in the model that $\hat{y}_t^j$ and $\hat{\pi}_t^j$ are the beginning-of-period forecasts of household j.

Proposition 3 (Hand-to-Mouth Posterior Beliefs)**.** *The unconditional correlation of output and inflation is positive iff*

$$C_{y,v}C_{\pi,v}\sigma_v^2 + C_{y,\gamma}C_{\pi,\gamma}\sigma_\gamma^2 > 0. \tag{26}$$

When $\chi_n \neq 0$, posterior beliefs of output and inflation are negatively correlated iff

$$\left(C_{y,v}\sigma_v^2 + \Xi C_{y,\gamma}\sigma_\gamma^2\right) \cdot \left(C_{\pi,v}\sigma_v^2 + \Xi C_{\pi,\gamma}\sigma_\gamma^2\right) < 0, \tag{27}$$

where $\Xi \equiv \frac{\chi_n(\tilde{\omega}_y C_{y,\gamma} - \tilde{\omega}_\gamma) - 1}{\chi_n \tilde{\omega}_y C_{y,v}}$. If $\chi_n = 0$, then (27) is equivalent to $C_{y,\gamma}C_{\pi,\gamma} < 0$.

The general conditions under which equations (26) and (27) hold are a function of the parameterization of the model, which are somewhat complicated. However, the following assumptions will help us derive more intuitive results.

Assumption 1. *Parameters are such that* $C_{y,v} > 0, C_{\pi,v} > 0, C_{y,\gamma} < 0, C_{\pi,\gamma} > 0$.

Assumption 1 implies that discount factor shocks v_t and wage cost-push shocks γ_t act like standard "aggregate demand" and "aggregate supply" shocks. This holds in the RANK version of the model, so for small enough λ will always be satisfied. However, this may fail if the feedback from the wage cost-push shock into aggregate output from the hand-to-mouth agents in (24) is large enough so that $C_{y,\gamma} > 0$.

The next corollary delivers two simple parameterizations which help deliver intuition regarding necessary and sufficient conditions for (27) to hold.

Corollary 3.1 (Hand-to-Mouth Posterior Beliefs)**.** *If Assumption 1 holds:*

(i) If $\chi_n = 0$*, then* (27) *is satisfied* $\forall \sigma_\gamma > 0$.

(ii) If $\chi_n \neq 0$*, then* $\exists\ \overline{\sigma_\gamma}$ *such that* $\sigma_\gamma < \overline{\sigma_\gamma}$ *implies that* (27) *does not hold.*

To understand case (i), note that whenever $\chi_n \approx 0$, fluctuations in the real wage have very small effects on the optimal labor decision. The natural benchmark case of log utility and no transfers satisfies this condition: the optimal labor choice is independent of the real wage due to offsetting income and substitution effects. More generally, $\chi_n \approx 0$ with $\varsigma \neq 1$ implies non-zero transfers which hedge H households from demand-driven movements in the real wage. Since firm profits are inversely related to labor costs (wages), a decline in labor income is offset by increased transfers. Case (i) thus implies that the optimal signal loads entirely on the aggregate wage cost-push shock γ_t. Because this signal contains no other information about realizations of other aggregate variables, posterior beliefs about all other outcomes are derived from the (conditional) response of the model to these shocks, so household posterior beliefs will always feature a negative correlation between output and inflation. For small enough values of cost-push shock volatility σ_γ^2, equation (26) will be satisfied, and actual inflation and output feature an unconditional positive correlation.

In case (ii), if the volatility of supply shocks are very small, then it is not optimal to dedicate much attention to these shocks. Instead (so long as $\chi_n \neq 0$), the optimal signal will place weight on the real wage, which in equilibrium will be driven more by discount factor (demand) shocks. Thus, posterior beliefs regarding output and inflation will be driven by the conditional response of the model to these demand shocks, implying a positive correlation in beliefs (as well as the data generating process).

4.2 Aggregate Responses

Next, we study the equilibrium effects of aggregate shocks. We are particularly focused on how the dynamics of our model differ from standard New Keynesian models. The following Proposition shows how the dynamics of our model depend on the amount of hand-to-mouth households and the degree of information frictions.

Proposition 4 (Aggregate Dynamics)**.** *In the limit of no hand-to-mouth agents ($\lambda \to 0$) and no information costs ($K \to 1$):*

(i) First derivatives of conditional responses with respect to the fraction of hand-to-mouth agents (λ) are

$$\frac{\partial C_{y,v}}{\partial \lambda} \to \frac{\varphi(1-\chi_n)}{(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2}, \quad \frac{\partial C_{y,\gamma}}{\partial \lambda} \to \frac{\varsigma\varphi(1-\chi_n)}{(\varsigma+\varphi)(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2},$$
$$\frac{\partial C_{\pi,v}}{\partial \lambda} \to \frac{(\varsigma+\varphi)\kappa_w\phi_\pi(1-\chi_n)}{(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2}, \quad \frac{\partial C_{\pi,\gamma}}{\partial \lambda} \to \frac{\varsigma\kappa_w\phi_\pi(1-\chi_n)}{(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2}.$$

(ii) Second derivatives of conditional responses with respect to the fraction of hand-to-mouth agents (λ) and information costs ($-K$) are

$$-\frac{\partial^2 C_{y,v}}{\partial \lambda \partial K} \to \frac{-\chi_n}{(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2}, \quad -\frac{\partial^2 C_{y,\gamma}}{\partial \lambda \partial K} \to \frac{\varsigma(1-\chi_n) + (\varsigma+\varphi)\kappa_w\phi_\pi}{(\varsigma + (\varsigma+\varphi)\kappa_w\phi_\pi)^2},$$
$$-\frac{\partial^2 C_{\pi,v}}{\partial \lambda \partial K} \to 0, \quad -\frac{\partial^2 C_{\pi,\gamma}}{\partial \lambda \partial K} \to 0.$$

In Proposition 4 we focus on the behavior of the model near a neighborhood of the full-information RANK benchmark. When $\lambda = 0$, Assumption 1 holds and the model behaves as expected: v_t and γ_t act as typical aggregate demand and supply shocks (where increases in either factor raise inflation; increases in v_t raise output, while increases in γ_t lower output). Result (i) shows that the existence of hand-to-mouth households effects how output responds to shocks. Whenever $\chi_n \leq 1$, hand-to-mouth agents *amplify* the output reaction to demand shocks, but *mitigate* the output reaction to supply shocks (and vice versa if $\chi_n \geq 1$).

The amplification of aggregate demand shocks is because the optimal consumption response of H households moves more than one-for-one with aggregate output whenever $\chi_n \leq 1$. The intuition is the same as Bilbiie (2020). The condition $\chi_n \leq 1$ is equivalent to the condition $\chi_c \geq 1$; this implies that all else equal, when aggregate income increases, H household consumption increases by more than S consumption, and H labor supply increases by less than S labor supply (or decreases by more if $\chi_n < 0$).

On the other hand, the same condition $\chi_n \leq 1$ implies supply shocks are mitigated. The reason is the following: a wage cost-push shock means that marginal costs for firms increases and therefore desired production falls. However, sticky prices imply that output falls by less than it otherwise would. Therefore, aggregate income is higher than would be otherwise, and so for the same reason as discussed previously, all else equal the H household consumption falls by less than the S households. Then the same income amplification channel in this context implies that in equilibrium output falls by less than the RANK benchmark.

The effects on inflation follow from the usual New Keynesian Phillips curve logic: inflation increases in response to higher marginal costs, which are a function of aggregate output. Inflation reactions to demand shocks are amplified if and only if the response of output is amplified. For wage cost-push shocks, the direct effect is to raise marginal costs, but this is dampened by the equilibrium decline in output. Thus, when the cost-push effects on output are mitigated, the equilibrium response of inflation is amplified.

The results above are when H households make optimal full-information choices. Result (ii) shows how the introduction of information costs changes the degree of amplification and mitigation discussed in (i).[13] We can think of the introduction of information costs as causing the H households to make mistakes when choosing labor supply. Note that the sign of $-\frac{\partial^2 C_{y,v}}{\partial \lambda \partial K}$ is determined by the sign of χ_n, not $1 - \chi_n$. Suppose $\chi_n < 0$. Then this result says that information costs lead to

[13]Note that $K = 1$ is equivalent to no costs of information ($\mu = 0$). The derivatives are evaluated with respect to $-K$, and so should be interpreted as the effect of increasing information costs. As shown in Section 3.1, in the case where agents receive a one-dimensional signal, the choice of Kalman gain K and the signal-to-noise ratio σ_η are inversely related, and both are monotonic functions of μ.

additional amplification of the output response to demand shocks. This may be surprising: typically, rational inattention models are characterized by *under*-reaction. The intuition for this result is as follows. In the FIRE-RANK limit, an increase in output increases the real wage. When $\chi_n < 0$, H households find it optimal to increase consumption but reduce labor when the real wage increases. Information frictions lead to an under-reaction of H labor supply to changes in the real wage. Because the H household labor supply mistake is supplying too much labor, H consumption over-reacts. The result follows from the amplification of demand shocks (since $\chi_n < 1$).

If $0 \leq \chi_n \leq 1$, then H households increase labor supply, but by less than under full information. Hence, H consumption also under-reacts. Thus, while TANK implies amplification, information costs weaken the amplification channel. Finally, if $\chi_n \geq 1$, we get the same under-reaction of both H consumption and labor supply. However, in this case TANK implies mitigation of the output reaction to demand shocks; thus, increasing information costs implies further mitigation.

Additionally, regardless of χ_n, for large enough ϕ_π we have that $-\frac{\partial^2 C_{y,\gamma}}{\partial\lambda\partial K} > 0$. The reason is that when ϕ_π is large enough, near the RANK limit the equilibrium optimal labor choice of H households is decreasing in γ_t (regardless of χ_n; even if $\chi_n \gg 0$, for large enough values of ϕ_π the equilibrium increase in wages will be small enough to imply a decline in optimal H labor choice). Under these conditions, an increase in information costs implies that actual H labor decisions under-react; that is, decline by less than the full-information benchmark. Thus, increasingly costly information implies more mitigation (if $\frac{\partial C_{y,\gamma}}{\partial\lambda} > 0$) or less amplification (if $\frac{\partial C_{y,\gamma}}{\partial\lambda} < 0$) of supply shocks relative to the full-information TANK model.

This result is reversed if ϕ_π is relatively small (and χ_n is large enough) so that the equilibrium hand-to-mouth response to a labor disutility shock is to *increase* labor supply. This will only occur if $\chi_n \gg 1$, which is possible only if transfers $\tau^D/\lambda > 1$.

Finally, we see that increasing information costs have no further effects on the transmission of shocks to inflation. This follows from two assumptions in the model. First, firm production is linear in labor (constant returns to scale); and second, the central bank only reacts to inflation when setting the policy rate. Intuitively, information costs cause the H households to make mistakes when choosing how much labor to supply; from the production function of firms, these labor supply mistakes are transmitted one-to-one (per unit of labor) to output. But this additional production is simply consumed by the H households, and thus in equilibrium does not lead to any changes in the pricing behavior of firms. The ensuing change in aggregate output does not affect the policy rate, and therefore does not change S household decisions.

Expectation Manipulation Policies: We utilize our model to explore the aggregate implications of policies which manipulate inflation expectations. Usually, such policies are considered in situations where a policymaker wishes to boost output today by raising inflation expectations. In models featuring FIRE, the only way in which a policymaker can manipulate expectations is by credibly committing to future policy actions. Without such future policy commitments, FIRE beliefs will be pinned down by the underlying dynamics of the model.

However, the existence of agents in our model with non-FIRE expectations potentially opens the door to other policies aimed at manipulating inflation expectations. We consider a policymaker who is able to manipulate the inflation expectations $E_t^j \pi_t$ of inattentive agents. We abstract from how the policymaker can manipulate the beliefs of the H households without taking any concrete policy actions. Instead, we assume such a policy is feasible, and use our model to study the aggregate consequences.

Formally, assume that the policymaker manipulates the average level of signals received by H households. The signal received by household j is now

$$s_t^j = n_t^{H,*} + \alpha z_t + \eta_t^j, \tag{28}$$

where z_t is common across all households $j \in [0, \lambda]$.[14] Choose $\alpha = \pm 1$ so that an increase in z_t is associated with an increase in inflation expectations: $\frac{\partial E_t^j \pi_t}{\partial z_t} > 0$. Will such a policy of manipulating inflation expectations lead to an increase in output? It turns out that the conditions under which this policy will fail are closely to tied to the conditions which lead to negatively correlated inflation and output beliefs, as shown in the following Proposition.

[14]Clearly a rationally inattentive agent would always choose a signal structure which puts zero weight on z_t whenever there is the possibility that $z_t \neq 0$. One way to generate this signal structure is to assume z_t has zero variance, so inattentive agents would be indifferent placing weight on z_t. In this interpretation, the policy should be thought of as a "one-off" (zero-probability) manipulation of inflation expectations.

Proposition 5 (Expectation Manipulation)**.** *Suppose H households receive the signal* (28)*. Then* $\frac{\partial y_t}{\partial z_t} > 0$ *iff* $C_{\pi,v}\sigma_v^2 + \Xi C_{\pi,\gamma}\sigma_\gamma^2 > 0$ *. If Assumption 1 holds and* $\chi_n = 0$*, then this condition is never satisfied.*

Proposition 5 provides a strong note of caution to policymakers: under "usual" parameter restrictions where inflation and output posterior beliefs are negatively correlated, the expectation manipulation policy will fail to boost output. Intuitively, the policy increases the inflation expectations by signalling to H households that a wage cost-push shock is likely. This causes H households to reduce labor supply and consumption. The firms and S household optimality conditions are unchanged (and firms face constant returns to scale), so this reduction in consumption translates into a one-to-one reduction in aggregate output.

5 Dynamic Model

Relaxing the assumption of iid shocks, we study the model with more complicated dynamics when $\rho_v \neq 0, \rho_\gamma \neq 0$. Aggregate H labor supply is given by $n_t^H = K n_t^{H,*} + (1-K)m_t$, where m_t are average priors of H households regarding the optimal labor decision (given by (23)). As discussed in Section 3.1, despite the simple AR(1) process for exogenous shocks, the dynamics of average priors m_t will in general be intractable. Fortunately, under the parametric assumption that $\chi_n = 0$, average priors evolve according to

$$m_t = \rho_\gamma (1-K) m_{t-1} - \rho_\gamma K \frac{1}{\varsigma + \varphi} \gamma_{t-1}. \tag{29}$$

The reason is that whenever $\chi_n = 0$, the optimal labor decision under full information is simply $n_t^{H,*} = -\frac{1}{\varsigma+\varphi}\gamma_t$. Thus, the Kalman updating process simply tracks an exogenous variable with known dynamics. The "infinite regress" problem only shows up if the optimal signal must track endogenous variables (such as the wage), which in equilibrium depend on the choices of other information-constrained agents.

Recall from (11) that $\chi_n = 0 \iff \varsigma^{-1} = 1 - \tau^D/\lambda$. While not without loss of generality, it nests the natural benchmark of log utility and no transfers. Thus, the gains in terms of tractability do not require unreasonable parametric assumptions.

Under the assumptions of AR(1) shocks and $\chi_n = 0$, we therefore have that $\mathbb{E}_t \Delta v_{t+1} = (\rho_v - 1)v_t$, $\mathbb{E}_t \Delta \gamma_{t+1} = (\rho_\gamma - 1)\gamma_t$, and $\mathbb{E}_t \Delta m_{t+1} = (\rho_\gamma(1-K)-1)m_t - \frac{\rho_\gamma K}{\varsigma+\varphi}\gamma_t$. Then the dynamics of output and inflation are function of v_t, γ_t and m_t:

$$\left(1 - \lambda\tilde{\zeta}_y\right)\mathbb{E}_t \Delta y_{t+1} = (1-\lambda)\varsigma^{-1}\left(i_t - \mathbb{E}_t \pi_{t+1} - v_t\right) + \lambda\left(\tilde{\zeta}_\gamma(\rho_\gamma - 1) - \zeta_n \rho_\gamma \frac{K}{\varsigma+\varphi}\right)\gamma_t + \lambda\zeta_n(\rho_\gamma(1-K)-1)m_t, \tag{30}$$

$$\pi_t = \kappa_w\left[\tilde{\omega}_\gamma \gamma_t + \tilde{\omega}_y y_t + \omega_n m_t\right] + \beta \mathbb{E}_t \pi_{t+1}, \tag{31}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t. \tag{32}$$

Equations (30) and (31) are somewhat complicated functions of the underlying parameters. However, the qualitative differences from the usual RANK model are apparent. First, the reaction of output to interest rate changes (and discount factor shocks) is not pinned down only by S household preferences (and similarly for inflation reactions to output and cost-push shocks). Second, output also reacts directly to wage cost-push shocks. Third, both output and inflation depend on the sluggish belief updating of information-constrained households.

5.1 Calibration

We choose the natural baseline of log utility ($\varsigma = 1$) and no transfers ($\tau^D/\lambda = 0$) which is consistent with $\chi_n = 0$. The discount factor is set to $\beta = 0.9975$, is consistent with an annualized long-run interest rate of approximately 4% (quarterly frequency). We choose standard Taylor rule coefficients of $\phi_\pi = 1.5$ and $\phi_y = 0.1$. In our baseline, we set the fraction of hand-to-mouth households $\lambda = 0.33$ (based on estimates in Kaplan et al. 2014 and Aguiar et al. 2024); Appendix Figures D3-D4 explores our model using alternative choices of λ. The remaining parameters are calibrated in order to match aggregate moments from U.S. data from 1978:Q1 to 2019:Q4.

We proxy y_t (output deviations from steady state) by the year-over-year growth rate in real GDP. For π_t (inflation deviations from steady state), we use the year-over-year growth rate of CPI. For w_t (wage deviations from steady state), we use the year-over-year growth rate in non-farm business sector unit labor costs. We choose unit labor costs because labor is the only productive input in our model, and aggregate cost-push shocks act through labor compensation. Data is from FRED (GDPC1, CPIAUCSL, ULCNFB) and detrended using the Hodrick-Prescott filter.

Finally, we utilize survey data as proxies for posterior beliefs of information-constrained agents. We continue to use the MSC because of the long sample period. However, most questions are qualitative and so are not well-suited to calibrating our quantitative model. Inflation expectations are one of the few exceptions recorded quantitatively, so we use this data to proxy for $\hat{\pi}_t^j$ (posterior beliefs of inflation deviations from steady state).

Table 4: Model Calibration

Parameter	Value	Description	Target
Panel A:			
β	0.9975	Discount Factor	Long-run rate
ς	1.0	CRRA	Log-utility, $\chi_n = 0$
$\frac{\tau^D}{\lambda}$	0.0	Transfers	Log-utility, $\chi_n = 0$
λ	0.33	Hand-to-Mouth	Fraction 1/3
ϕ_π	1.5	Taylor Rule	Inflation Coeff.
ϕ_y	0.1	Taylor Rule	Output Coeff.
Panel B:			
φ	0.5301	$\sigma(w_t)$	1.5682
κ_w	0.198	$\rho(y_t, \pi_t)$	0.0689
ρ_v	0.7133	$\rho(y_t, y_{t-1})$	0.8074
ρ_γ	0.8239	$\rho(\pi_t, \pi_{t-1})$	0.749
σ_v	0.7613	$\sigma(y_t)$	1.5757
σ_γ	1.7843	$\sigma(\pi_t)$	1.2007
K	0.151	$\rho\left(\hat{\pi}_t^j, \pi_t\right)$	0.332

Notes: Panel A reports parameters set to standard values. Panel B reports our parameters which are calibrated to match empirical moments. For each parameter in Panel B, we include the moment which is most closely related; however, the calibration exercise jointly determines the parameter values.

Table 4 summarizes our calibration. We jointly calibrate the remaining model parameters by targeting second moments in the data. First, we target the volatility of y_t, π_t, and w_t as well as the (quarterly) autocorrelation of y_t and π_t. These moments are informative about parameters in the model which govern the volatility and persistence of shocks (σ_v, σ_γ, ρ_v, ρ_γ), as well as the inverse Frisch elasticity (φ). Additionally, we target the correlation of y_t and π_t (which is informative about the slope of the Phillips curve κ_w). We also target the correlation of $\hat{\pi}_t^j$ and π_t (which is informative about information frictions K).

Our parameter estimates are broadly in line with typical calibrations used in the

New Keynesian literature. Our calibration implies that our supply factor shocks are more persistent than demand shocks, but that they are jointly consistent with a weak positive correlation of output and inflation over the sample. We find a somewhat large Frisch elasticity: our estimate $\varphi^{-1} > 1$. This is in line with the evidence from the "macro" literature of wage elasticities, but contrasts with "micro" estimates.

We estimate a large degree of information frictions due to the overall low degree of correlation between inflation beliefs across households and actual inflation. Although inflation expectations are well-suited for estimating information costs in the model, Appendix D re-estimates the model across a range of values $K \in (0, 1)$. Appendix Figure D1 shows that most of the parameters are relatively insensitive to the degree of information frictions. Appendix Figure D2 shows that beliefs regarding output and inflation remain negatively correlated even for very low information costs.

5.2 Dynamic Responses

Using the calibrated model, we explore the response to demand and supply shocks. We consider two initial conditions: starting from steady state, and an alternative where H household priors m_t are "low" (two standard deviations below steady state).

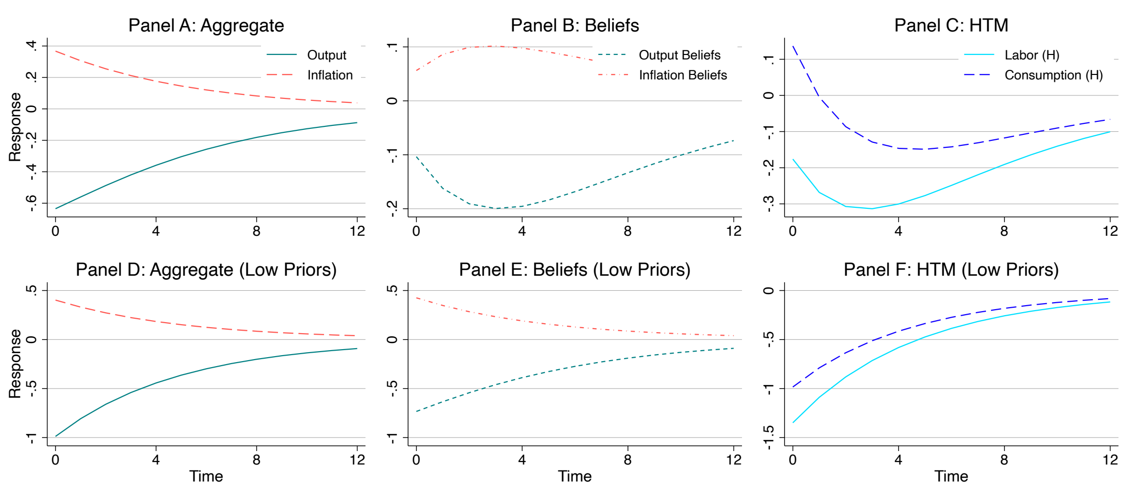

Figure 3: Response to Supply Shock

Notes: IRFs following an increase in the wage cost-push shock. The first column reports aggregate output and inflation; the second column reports average H beliefs; the third column reports average H labor and consumption. Each row corresponds to different initial conditions regarding H household priors (steady state or low, respectively).

Figure 3 reports model IRFs following a standard deviation shock to the supply factor innovation $\varepsilon_{t,\gamma}$. Each row corresponds to different initial conditions regarding H household priors (steady state or low, respectively). Focusing on the first row, Panel A reports the dynamics of output (solid line) and inflation (dashed line) following the shock. On impact, the supply factor shock leads to a fall in output and an increase in inflation. Inflation then falls towards steady state, while output increases back towards steady state as the shock dissipates.

Panel B reports how H household beliefs regarding output and inflation react to the shock.[15] As discussed above, H households learn about supply shocks, and thus the increase in γ_t leads to H households to update their beliefs. Although quantitatively different from the realizations of output and inflation, output and inflation beliefs move in line with realizations. However, H household actions differ from the full-information benchmark. As shown in Panel C, household labor supply declines following the shock (as shown in the solid line), but it under-reacts relative to a full-information baseline, which implies that H consumption actually increases (as shown in the dashed line). As additional information is collected in the next period, H labor supply on average declines even further (at which point H consumption also drops below steady state). The sluggish reaction to information leads to aggregate hump-shaped movements in H household actions.

Panels D, E, and F report the same responses when initial priors m_t are low. In this case, H households ex-ante believe that the optimal labor supply decision is below steady state because the supply factor is high. Thus, H households on average already believe that the likelihood of being in a supply-driven recession is large. Thus, on impact the supply shock leads to a larger decline in output driven by the larger decline in H household labor supply. In this case, H household consumption is initially below steady state, and decreases even further as more information is collected. All else equal, the larger fall in output puts downward pressure on the policy rate, and thus inflation rises by more than in the case where m_t is at steady state.

Figure 4 conducts the same set of experiments, but following a standard deviation shock to the demand factor innovation $\varepsilon_{t,v}$. As shown in Panel A, on impact the demand factor shock leads to a boost in output and an increase in inflation. Both

[15]We focus on posterior beliefs (beginning-of-period forecasts) for simplicity. The dynamics of 1-year ahead inflation and output forecasts (as well as state variables) are in Appendix Figures D5-D6.

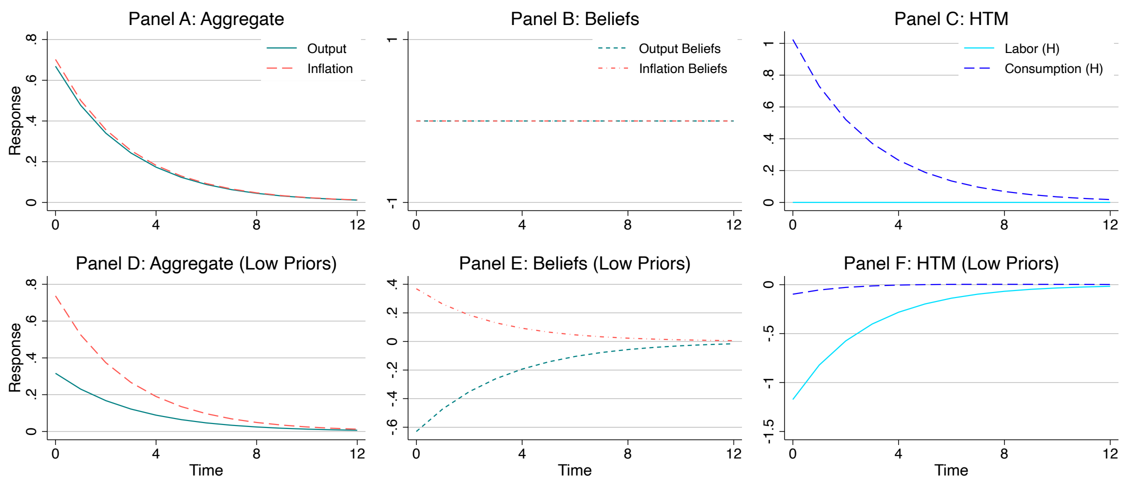

Figure 4: Response to Demand Shock

Notes: IRFs following an increase in the discount factor shock. The first column reports aggregate output and inflation; the second column reports average H beliefs; the third column reports average H labor and consumption. Each row corresponds to different initial conditions regarding H household priors (steady state or low, respectively).

output and inflation then monotonically decline towards steady state as the shock dissipates. Panel B reports how H household beliefs regarding output and inflation react to the shock. As discussed above, H household signals load entirely on supply shocks. Thus, following a demand shock when H household priors are at steady state, average beliefs do not react at all. This implies that H households on average do not adjust their labor supply (as shown in the solid line in Panel C). Due to the increase in wages following the demand-driven expansion, H household consumption on average therefore increases (as shown in the dashed line in Panel C).

Panels D, E, and F report the same responses when initial priors m_t are low. In this case, H households ex-ante believe that the likelihood of entering a supply-driven recession is high and that the optimal labor supply decision is below steady state. H households therefore initially reflect this belief: inflation expectations are high and output beliefs are low. Because of this, H household labor supply is reduced relative to the previous case, and H consumption also declines. Thus, while the response to the demand shock is expansionary, the initial low H priors imply the expansion is smaller than otherwise. As time passes, while the H households do not learn about the level of aggregate demand, their signals are consistent with the supply factor being at steady state, and thus H households sluggishly update their beliefs towards steady state.

Expectation Manipulation Dynamics: Figure 5 repeats the expectation policy experiment where the central bank increases inflation expectations. H households conclude that inflation is high due to a supply-driven recession. Therefore, output beliefs fall and these households decrease their labor supply. This implies a nearly one-for-one reduction in consumption. Thus, aggregate output declines. Since aggregate output falls, a cut in the policy rate puts upward pressure on inflation. Inflation therefore rises on impact, before subsiding as beliefs return to steady state.

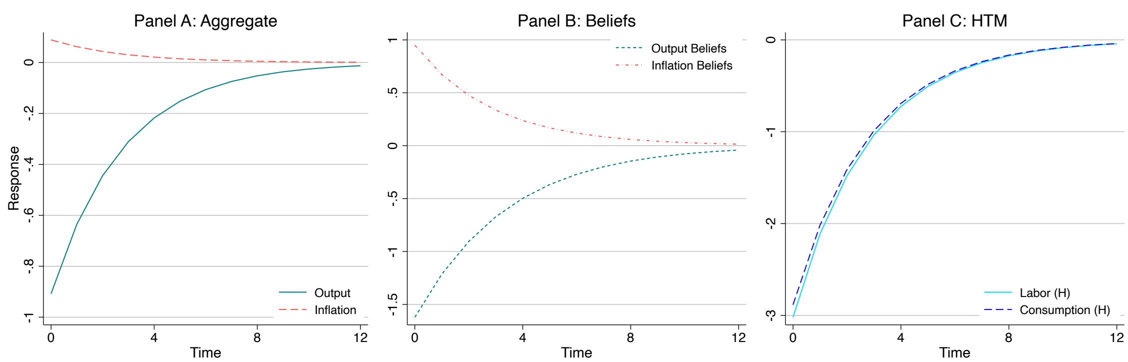

Figure 5: Response to Expectation Shock

Notes: IRFs following an expectation manipulation policy shock. Panel A reports aggregate output and inflation; Panel B reports average H beliefs; Panel C reports average H labor and consumption.

Note that the response of inflation is relatively small: output declines by 0.9% but inflation only increases by less than 0.1%. The decisions of S households (with FIRE beliefs) are only affected through changes in the policy rate; because these are small, changes in equilibrium wages are also small. Thus, the implied increase in consumption from H households is produced nearly one-for-one from the increase in H labor supply.[16]

6 Concluding Remarks

Consumer beliefs about aggregate and personal economic conditions exhibit a low-dimension factor structure. One single component drives the vast majority of fluctu-

[16] As discussed above, if firm production features decreasing returns to scale, inflation will react more strongly and the quantitative aggregate responses will differ. However, the qualitative reaction of output and H household decisions are similar. In particular, the central bank's policy of raising inflation expectations is counterproductive and results in a decline in aggregate output.

ations in beliefs, and this factor seemingly acts like "sentiment." We rationalize this puzzling behavior in a New Keynesian model featuring rational inattention. Agents economize on information costs by obtaining a signal about a combination of both supply and demand shocks. For consumers relying on labor income, this information acquisition strategy typically implies higher precision in beliefs about supply-driven recessions and less about demand-driven recessions; thus, belief correlations differ in sign from the underlying data-generating process. The model shows the manner in which inattentive agents reduce the dimensionality of the problem; why they choose to learn about one component; how this leads to a counter-intuitive correlation of expectations in the cross-section; and how the aggregate dynamics of the model are affected by information frictions. Our findings provide a note of caution for central bankers considering policies which manipulate inflation expectations: raising inflation expectations can easily backfire, leading to further recessionary pressures.

Appendix A Proofs

Proof of Proposition 1. Assuming Blanchard and Kahn (1980) determinacy conditions are met in (16), the unique rational expectations equilibrium is given by

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{C}_x \boldsymbol{\varepsilon}_t, \quad \mathbf{y}_t = \mathbf{A}_y \mathbf{x}_{t-1} + \mathbf{C}_y \boldsymbol{\varepsilon}_t. \tag{A1}$$

The dynamics matrices follow from the usual partitioning of the eigendecomposition $\tilde{\mathbf{A}} = \hat{\mathbf{V}}_t \mathbf{J}_t \mathbf{V}_t$: the diagonal matrices $\mathbf{J}_1, \mathbf{J}_2$ collect all eigenvalues inside and outside of the unit circle, respectively. Partition the matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{C}}, \hat{\mathbf{V}}_t, \mathbf{V}_t$ accordingly, so the dynamics matrices in equations (A1) are given by $\mathbf{A}_y \equiv -\mathbf{V}_{22}^{-1}\mathbf{V}_{21}$, $\mathbf{C}_y \equiv -\mathbf{V}_{22}^{-1}\mathbf{J}_2^{-1}\left[\mathbf{V}_{21}\tilde{\mathbf{C}}_1 + \mathbf{V}_{22}\tilde{\mathbf{C}}_2\right]$, $\mathbf{A}_x \equiv \tilde{\mathbf{A}}_{11} + \tilde{\mathbf{A}}_{12}\mathbf{A}_y$, and $\mathbf{C}_x \equiv \tilde{\mathbf{C}}_1 + \tilde{\mathbf{A}}_{12}\mathbf{C}_y$. Hence, both $\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t$ collected into the vector $\mathbf{X}_t$ evolve jointly according to (17). Then given a (time-invariant) signal of the form in (19), Kalman updating implies that (time-invariant) prior and posterior covariance matrices solve

$$\boldsymbol{\Sigma}_{1|1} = (\mathbf{I} - \mathbf{K}\mathbf{H})\boldsymbol{\Sigma}_{1|0}, \quad \boldsymbol{\Sigma}_{1|0} = \mathbf{A}\boldsymbol{\Sigma}_{1|1}\mathbf{A}^\top + \mathbf{C}\mathbf{C}^\top, \quad \mathbf{K} = \boldsymbol{\Sigma}_{1|0}\mathbf{H}^\top\left(\mathbf{H}\boldsymbol{\Sigma}_{1|0}\mathbf{H}^\top + \boldsymbol{\Sigma}_\eta\right)^{-1}, \tag{A2}$$

and state variables and posterior beliefs jointly evolve according to

$$\begin{bmatrix}\mathbf{X}_t\\ \hat{\mathbf{X}}_t^j\end{bmatrix}=\mathcal{A}\begin{bmatrix}\mathbf{X}_{t-1}\\ \hat{\mathbf{X}}_{t-1}^j\end{bmatrix}+\mathcal{C}\begin{bmatrix}\boldsymbol{\varepsilon}_t\\ \boldsymbol{\eta}_t^j\end{bmatrix},\quad \mathcal{A}\equiv\begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{KHA} & (\mathbf{I}-\mathbf{KH})\,\mathbf{A}\end{bmatrix},\quad \mathcal{C}\equiv\begin{bmatrix}\mathbf{C} & \mathbf{0}\\ \mathbf{KHC} & \mathbf{K}\end{bmatrix}. \tag{A3}$$

Given per-period quadratic utility in equation (18), certainty equivalence implies that the optimal action of agent j is a function of posterior beliefs: $\mathbf{a}_t^j \equiv -\frac{1}{2}\mathbf{B}_{aa}^{-1}\mathbf{B}_{ax}^\top\hat{\mathbf{X}}_t^j$. Then the problem of agent j is equivalent to minimizing losses due to misperceptions. Since $\beta^j = 0$, the problem is equivalent to the following repeated static optimization problem. Given a prior covariance $\boldsymbol{\Sigma}_{1|0}$ and defining $\boldsymbol{\Omega}$ as in (21), the problem is equivalent to picking a posterior covariance $\boldsymbol{\Sigma}$ to solve minimize $\operatorname{Tr}\boldsymbol{\Omega}\boldsymbol{\Sigma} + \frac{1}{2}\mu\left(\operatorname{logdet}\boldsymbol{\Sigma}_{1|0} - \operatorname{logdet}\boldsymbol{\Sigma}\right)$, subject to the no-forgetting constraint $\mathbf{0} \preccurlyeq \boldsymbol{\Sigma} \preccurlyeq \boldsymbol{\Sigma}_{1|0}$ (as in Kőszegi and Matějka (2020) and Miao et al. (2022)). Using the cyclical properties of the trace operator, this is equivalent to solving

$$\min_{\tilde{\boldsymbol{\Sigma}}} \operatorname{Tr}\tilde{\boldsymbol{\Omega}}\tilde{\boldsymbol{\Sigma}} - \frac{1}{2}\mu\operatorname{logdet}\tilde{\boldsymbol{\Sigma}},\quad \text{s.t. } \mathbf{0}\preccurlyeq\tilde{\boldsymbol{\Sigma}}\preccurlyeq\mathbf{I}, \tag{A4}$$

where $\tilde{\boldsymbol{\Omega}} = \boldsymbol{\Sigma}_{1|0}^{1/2}\boldsymbol{\Omega}\boldsymbol{\Sigma}_{1|0}^{1/2}$ $\tilde{\boldsymbol{\Sigma}} = \boldsymbol{\Sigma}_{1|0}^{-1/2}\boldsymbol{\Sigma}\boldsymbol{\Sigma}_{1|0}^{-1/2}$ (note Appendix C.2 shows that whenever $\mu > 0$, $\boldsymbol{\Sigma}_{1|0}^{-1/2}$ exists). The first-order conditions are therefore the same as in Kőszegi and Matějka (2020); the result follows.

For H households, $\operatorname{rank}\boldsymbol{\Omega} = 1$ (see Appendix C.3); Corollary 1.1 follows. □

Proof of Proposition 2. From equation (A3), long-run covariances of state and jump variables and posterior beliefs solve the following Lyapunov equation

$$\mathbb{V}ar\begin{bmatrix}\mathbf{X}_t\\ \hat{\mathbf{X}}_t\end{bmatrix}\equiv\mathcal{S}=\mathcal{A}\mathcal{S}\mathcal{A}^\top+\mathcal{C}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Sigma}_\eta\end{bmatrix}\mathcal{C}^\top, \tag{A5}$$

and $\boldsymbol{\Sigma}_y = \begin{bmatrix}\mathbf{A}_y & \mathbf{C}_y\end{bmatrix}\boldsymbol{\Sigma}_X\begin{bmatrix}\mathbf{A}_y^\top\\ \mathbf{C}_y^\top\end{bmatrix}$, $\boldsymbol{\Sigma}_{\hat{y}} = \begin{bmatrix}\mathbf{A}_y & \mathbf{C}_y\end{bmatrix}\boldsymbol{\Sigma}_{\hat{X}}\begin{bmatrix}\mathbf{A}_y^\top\\ \mathbf{C}_y^\top\end{bmatrix}$. From (A3), unless $\boldsymbol{\Sigma}_\eta = \mathbf{0}$ and $\mathbf{KH} = \mathbf{I}$ (when all information frictions are eliminated so posterior beliefs $\hat{\mathbf{X}}_t^j \equiv \mathbf{X}_t$), we have $\boldsymbol{\Sigma}_X \neq \boldsymbol{\Sigma}_{\hat{X}}$ which also implies $\boldsymbol{\Sigma}_y \neq \boldsymbol{\Sigma}_{\hat{y}}$.

Let $N_1 \equiv \operatorname{rank}\boldsymbol{\Omega}$ and $N \equiv \dim\mathbf{X}_t$. First, from the proof of Prop. 1, the optimal signal structure implies $\mathbf{H}\boldsymbol{\Sigma}_{1|0}\mathbf{H}^\top = \mathbf{I}_1$ (the identity matrix with dimension equal to N_1). Thus, $\mathbf{HK} = (\mathbf{I}_1 + \boldsymbol{\Sigma}_\eta)^{-1}$ and $\mathbf{KH} = \boldsymbol{\Sigma}_{1|0}^{1/2}\mathbf{U}_1 (\mathbf{I}_1 + \boldsymbol{\Sigma}_\eta)^{-1} \mathbf{U}_1^\top \boldsymbol{\Sigma}_{1|0}^{-1/2}$. Thus we have that $\mathbf{KH} = \mathbf{I}$ (the identity matrix with dimension equal to J) iff $\boldsymbol{\Sigma}_\eta = \mathbf{0}$ and $\mathbf{U}_1\mathbf{U}_1^\top = \mathbf{I}$. The former condition does not hold whenever $\mu > 0$ (information costs are non-zero); the latter condition does not hold whenever $N_1 < N$ ($\boldsymbol{\Omega}$ is not full rank).

From equation (A5), posterior conditional covariances are $\check{\boldsymbol{\Sigma}}_{\hat{X}} = \mathcal{C} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_\eta \end{bmatrix} \mathcal{C}^\top$. Since these covariances condition on $\mathbf{X}_{t-1}, \hat{\mathbf{X}}_{t-1}^j$, $\check{\boldsymbol{\Sigma}}_{\hat{X}}$ is all zeros besides the bottom-right block given by $\mathbf{K}\left[\mathbf{H}_\epsilon \mathbf{C}_x \mathbf{C}_x^\top \mathbf{H}_\epsilon^\top + \boldsymbol{\Sigma}_\eta\right]\mathbf{K}^\top$. This implies that jump posterior conditional covariances $\check{\boldsymbol{\Sigma}}_{\hat{y}} = \mathbf{C}_y\mathbf{K}\left[\mathbf{H}_\epsilon \mathbf{C}_x \mathbf{C}_x^\top \mathbf{H}_\epsilon^\top + \boldsymbol{\Sigma}_\eta\right]\mathbf{K}^\top\mathbf{C}_y^\top$. Because $\operatorname{rank}\mathbf{K} \leq N_1$, we also have that $\operatorname{rank}\check{\boldsymbol{\Sigma}}_{\hat{X}} \leq N_1$ and $\operatorname{rank}\check{\boldsymbol{\Sigma}}_{\hat{y}} \leq N_1$ for any $\mu > 0$.

Finally, when $\mathbf{A}_x = \mathbf{0}$, the state space is $\mathbf{X}_t \equiv \boldsymbol{\varepsilon}_t$, and further, $\boldsymbol{\Sigma}_{1|0} = \mathbf{CC}^\top \equiv \mathbf{I}$. Thus when $N_1 = 1$ so the signal coefficient vector $\mathbf{h} \equiv \mathbf{u}_1^\top$ is one-dimensional, we have that $\boldsymbol{\Sigma}_{\hat{y}} = \check{\boldsymbol{\Sigma}}_{\hat{y}} \propto \mathbf{C}_y\mathbf{K}\mathbf{K}^\top\mathbf{C}_y^\top = \mathbf{C}_y\mathbf{h}^\top\mathbf{h}\mathbf{C}_y^\top$ (which follows because the gain vector $\mathbf{K} = \mathbf{h}^\top \cdot (1+\sigma_\eta^2)^{-1}$ is also one-dimensional). Any aggregate variable y_t can be written as a linear combination $\mathbf{cX}_t$ for some row vector $\mathbf{c}$, and posterior beliefs also satisfy $\hat{y}_t^j = \mathbf{c}\hat{\mathbf{X}}_t^j$. Since $\mathbb{C}ov\left(\mathbf{cX}_t, \mathbf{hX}_t\right) = \mathbf{c}\boldsymbol{\Sigma}_{1|0}\mathbf{h}^\top$, this implies $\mathbb{C}ov\left(\mathbf{c}_1\hat{\mathbf{X}}_t^j, \mathbf{c}_2\hat{\mathbf{X}}_t^j\right) \propto \mathbf{c}_1\boldsymbol{\Sigma}_{1|0}\mathbf{h}^\top\mathbf{h}\boldsymbol{\Sigma}_{1|0}\mathbf{c}_2^\top = \mathbb{C}ov\left(\mathbf{c}_1\mathbf{X}_t, \mathbf{hX}_t\right) \cdot \mathbb{C}ov\left(\mathbf{c}_2\mathbf{X}_t, \mathbf{hX}_t\right)$. Then the results of Corollary 2.1 follow setting $\mathbf{h} = \mathbf{e}_k^\top$ for (i) and setting $\mathbf{c}_1 = \mathbf{h}$ for (ii). □

Proof of Proposition 3. Computing $\mathbb{C}ov(y_t, \pi_t)$ using equations (25) gives (26). From the proof of Prop. 2, we have that $\mathbb{C}ov(\hat{y}_t^j, \hat{\pi}_t^j) \propto \mathbb{C}ov(y_t, n_t^{H,*}) \cdot \mathbb{C}ov(\pi_t, n_t^{H,*})$. These covariances are given by (27), which follows since with iid dynamics, $n_t^{H,*} \propto \chi_n\tilde{\omega}_y y_t + (\chi_n\tilde{\omega}_\gamma - 1)\gamma_t$ (using the derivations in equations (C5)-(C12)).

If $\chi_n = 0$, then $n_t^{H,*} \propto -\gamma_t$ and so $\mathbb{C}ov(\hat{y}_t^j, \hat{\pi}_t^j) \propto C_{y,\gamma}C_{\pi,\gamma}\sigma_\gamma^2$. Hence if $C_{y,\gamma} < 0$ and $C_{\pi,\gamma} > 0$, $\mathbb{C}ov(\hat{y}_t^j, \hat{\pi}_t^j) < 0$ iff $\sigma_\gamma^2 > 0$. If $\chi_n \neq 0$, then if $\sigma_\gamma^2 = 0$, $\mathbb{C}ov(\hat{y}_t^j, \hat{\pi}_t^j) \propto C_{y,v}C_{\pi_v}\sigma_v^2$, which is positive if $C_{y,v} > 0$ and $C_{\pi,v} > 0$. Corollary 3.1 follows. □

Proof of Proposition 4. Taking first- and second- derivatives of (C5)-(C8) with respect to λ and $-K$ and evaluating at $\lambda = 0, K = 1$ gives results (i) and (ii). □

Proof of Proposition 5. The policy boosts output $\frac{\partial y_t}{\partial z_t} > 0 \iff \frac{\partial n_t^H}{\partial z_t} > 0$. By assumption, $\frac{\partial \pi_t^j}{\partial z_t} > 0$. From the proof of Prop. 2, $\frac{\partial n_t^H}{\partial z_t} > 0 \iff \mathbb{C}ov(\pi_t, n_t^{H,*}) > 0 \iff C_{\pi,v}\sigma_v^2 + \Xi C_{\pi,\gamma}\sigma_\gamma^2 > 0$, which follows from the proof of Prop. 3. □

References

Afrouzi, H. and Yang, C. (2021). Dynamic Rational Inattention and the Phillips Curve. Technical report.

Aguiar, M., Bils, M., and Boar, C. (2024). Who Are the Hand-to-Mouth? Forthcoming, *Review of Economic Studies*.

Andre, P., Haaland, I., Roth, C., and Wohlfart, J. (2023). Narratives About the Macroeconomy. CESifo Working Paper Series 10535, CESifo.

Andre, P., Pizzinelli, C., Roth, C., and Wohlfart, J. (2022). Subjective Models of the Macroeconomy: Evidence from Experts and Representative Samples. *The Review of Economic Studies*, 89(6):2958–2991.

Angeletos, G.-M. and La'O, J. (2013). Sentiments. *Econometrica*, 81(2):739–779.

Bhandari, A., Borovička, J., and Ho, P. (2024). Survey Data and Subjective Beliefs in Business Cycle Models. Forthcoming, *Review of Economic Studies*.

Bilbiie, F. O. (2020). The New Keynesian Cross. *Journal of Monetary Economics*, 114(C):90–108.

Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311.

Bordalo, P., Gennaioli, N., Ma, Y., and Shleifer, A. (2020). Overreaction in macroeconomic expectations. *American Economic Review*, 110(9):2748–82.

Bordalo, P., Gennaioli, N., Porta, R. L., O'Brien, M., and Shleifer, A. (2023). Long Term Expectations and Aggregate Fluctuations. In *NBER Macroeconomics Annual 2023, volume 38*, NBER Chapters.

Candia, B., Coibion, O., and Gorodnichenko, Y. (2021). The Inflation Expectations of U.S. Firms: Evidence from a new survey. NBER Working Papers 28836.

Carroll, C. D. (2003). Macroeconomic Expectations of Households and Professional Forecasters. *The Quarterly Journal of Economics*, 118(1):269–298.

Carvalho, C. and Nechio, F. (2014). Do People Understand Monetary Policy? *Journal of Monetary Economics*, 66(C):108–123.

Coibion, O., Gorodnichenko, Y., and Kamdar, R. (2018a). The Formation of Expectations, Inflation, and the Phillips Curve. *Journal of Economic Literature*, 56(4):1447–1491.

Coibion, O., Gorodnichenko, Y., and Kumar, S. (2018b). How Do Firms Form Their Expectations? New Survey Evidence. *American Economic Review*, 108(9):2671–2713.

Coibion, O., Gorodnichenko, Y., Kumar, S., and Pedemonte, M. (2020a). Inflation expectations as a policy tool? *Journal of International Economics*, 124:103297.

Coibion, O., Gorodnichenko, Y., and Ropele, T. (2020b). Inflation Expectations and Firm Decisions: New Causal Evidence. *The Quarterly Journal of Economics*, 135(1):165–219.

Dräger, L., Lamla, M. J., and Pfajfar, D. (2016). Are Survey Expectations Theory-Consistent? The Role of Central Bank Communication and News. *European Economic Review*, 85(C):84–111.

Galí, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press.

Hajdini, I., Knotek, E. S., Leer, J., Pedemonte, M., Rich, R. W., and Schoenle, R. (2022). Low Passthrough from Inflation Expectations to Income Growth Expectations: Why People Dislike Inflation. Working Papers 22-21R, Federal Reserve Bank of Cleveland.

Jiang, J. H., Kamdar, R., Lu, K., and Puzzello, D. (2024). How do Households Respond to Expected Inflation? An Investigation of Transmission Mechanisms. Working paper.

Kaplan, G., Violante, G. L., and Weidner, J. (2014). The Wealthy Hand-to-Mouth. *Brookings Papers on Economic Activity*, 45(1 (Spring):77–153.

Kőszegi, B. and Matějka, F. (2020). Choice Simplification: A Theory of Mental Budgeting and Naive Diversification. *The Quarterly Journal of Economics*, 135(2):11531207.

Maćkowiak, B., Matějka, F., and Wiederholt, M. (2018). Dynamic Rational Inattention: Analytical Results. *Journal of Economic Theory*, 176(C):650–692.

Maćkowiak, B. and Wiederholt, M. (2009). Optimal Sticky Prices under Rational Inattention. *American Economic Review*, 99(3):769–803.

Maćkowiak, B. and Wiederholt, M. (2015). Business Cycle Dynamics under Rational Inattention. *Review of Economic Studies*, 82(4):1502–1532.

Malmendier, U. and Nagel, S. (2016). Learning from Inflation Experiences. *The Quarterly Journal of Economics*, 131(1):53–87.

Mani, A., Mullainathan, S., Shafir, E., and Zhao, J. (2013). Poverty Impedes Cognitive Function. *Science*, 341(6149):976–980.

Mankiw, N. G. (2000). The Savers-Spenders Theory of Fiscal Policy. *American Economic Review*, 90(2):120–125.

Mankiw, N. G. and Reis, R. (2002). Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, 117(4):1295–1328.

McKay, A., Nakamura, E., and Steinsson, J. (2016). The Power of Forward Guidance Revisited. *American Economic Review*, 106(10):3133–3158.

Miao, J., Wu, J., and Young, E. R. (2022). Multivariate Rational Inattention. *Econometrica*, 90(2):907–945.

Sergeyev, D., Lian, C., and Gorodnichenko, Y. (2023). The Economics of Financial Stress. Technical report, National Bureau of Economic Research.

Shapiro, A. H., Sudhof, M., and Wilson, D. J. (2022). Measuring News Sentiment. *Journal of Econometrics*, 228(2):221–243.

Shiller, R. J. (1996). Why Do People Dislike Inflation? Cowles Foundation Discussion Papers 1115, Cowles Foundation for Research in Economics, Yale University.

Sims, C. A. (2003). Implications of Rational Inattention. *Journal of Monetary Economics*, 50(3):665–690.

Stantcheva, S. (2024). Why Do We Dislike Inflation? *Brookings Papers on Economic Activity*.

Online Appendix for
"Attention-Driven Sentiment and the Business Cycle"

by Rupal Kamdar and Walker Ray

Appendix B Additional Figures and Tables

Table B1: Inflation/Unemployment Regressions (SCE)

Panel A:		Pooled	
	(1)	(2)	(3)
Unemployment Up	1.903***	1.024***	0.401***
	(0.107)	(0.105)	(0.102)
Unemployment Down	-1.567***	-0.777***	-0.280***
	(0.068)	(0.072)	(0.071)
FEs	N	Y	Y
R-sq	0.029	0.467	0.537
Obs.	150,578	147,082	129,860
Panel B:		Education	
	(1)	(2)	(3)
Unemployment Up	1.904***	1.439***	0.562***
	(0.510)	(0.212)	(0.102)
Unemployment Down	-2.093***	-0.888***	-0.393***
	(0.310)	(0.148)	(0.074)
FEs	Y	Y	Y
R-sq	0.448	0.482	0.438
Obs.	16,636	47,639	82,419
Panel C:		Income	
	(1)	(2)	(3)
Unemployment Up	1.524***	1.134***	0.276**
	(0.226)	(0.166)	(0.125)
Unemployment Down	-1.210***	-0.564***	-0.422***
	(0.155)	(0.111)	(0.091)
FEs	Y	Y	Y
R-sq	0.469	0.451	0.439
Obs.	50,551	51,979	43,086
Panel D:		Age	
	(1)	(2)	(3)
Unemployment Up	0.817***	1.560***	0.543***
	(0.198)	(0.174)	(0.175)
Unemployment Down	-0.697***	-0.735***	-0.887***
	(0.147)	(0.118)	(0.116)
FEs	Y	Y	Y
R-sq	0.426	0.500	0.462
Obs.	41,917	57,304	47,801

Notes: estimates of equation (1) using the SCE. We define "unemployment up/down" indicator variables as a function of responses regarding the probability of an increase in unemployment in the following year. We use 60%-100% for "up" and 0%-40% for "down." Panel A is pooled across all consumers. Each column of Panel B restricts to consumers with different education attainment: high school (1); some college (2); or a college degree (3). Panel C estimates separately across the income distribution: under 50k (1); 50k-100k (2); or over 100k (3). Panel D estimates the regression separately for different age groups: under 40 (1); between 40 and 60 (2); and over 60 (3).

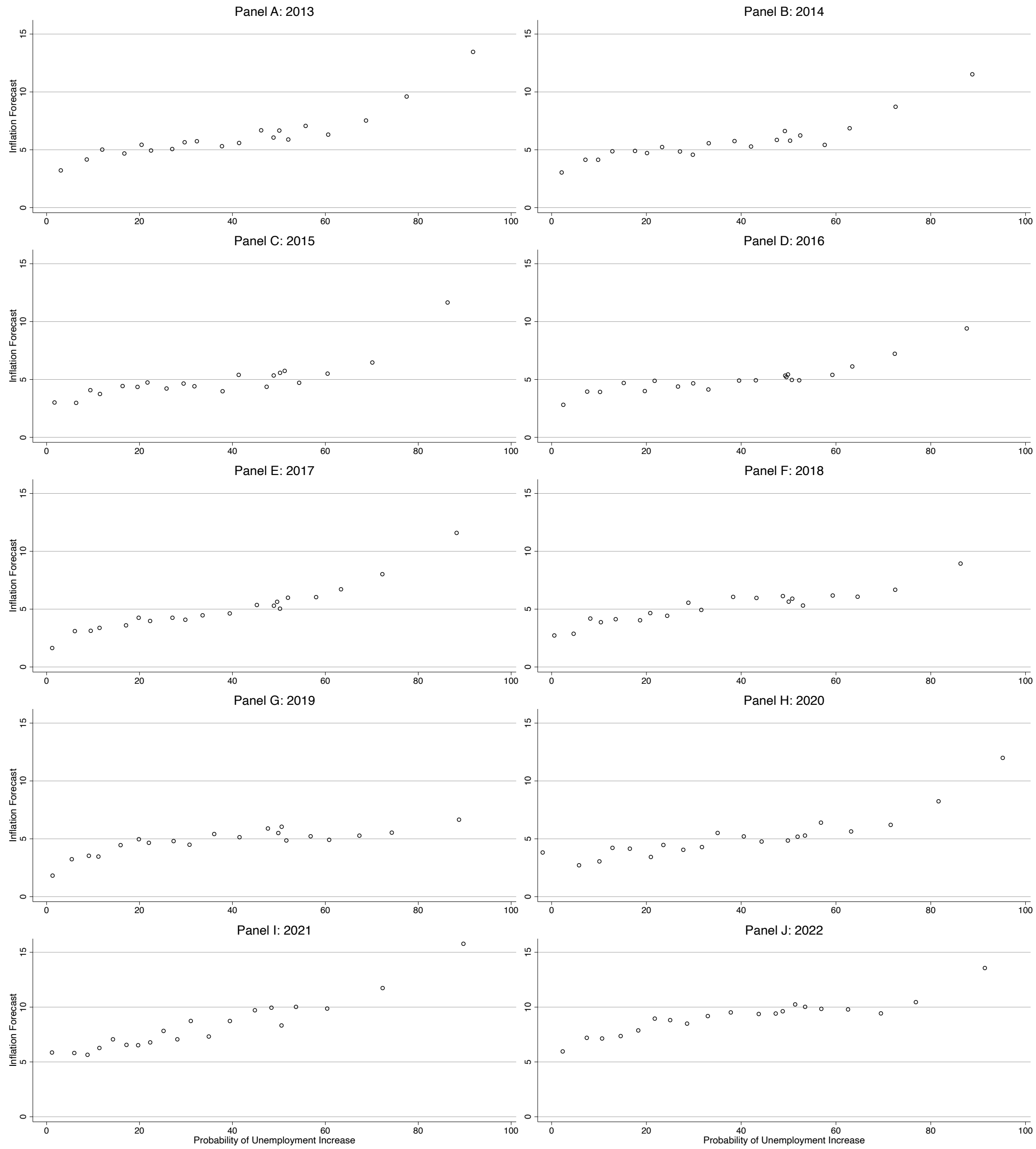

Figure B1: SCE Inflation/Unemployment Binscatter Plots

Notes: binscatter plots of inflation expectations (y-axis) and the probability of an increase in unemployment in the following year (x-axis). Each panel plots the binscatter from respondents in a given year (from 2013-2022).

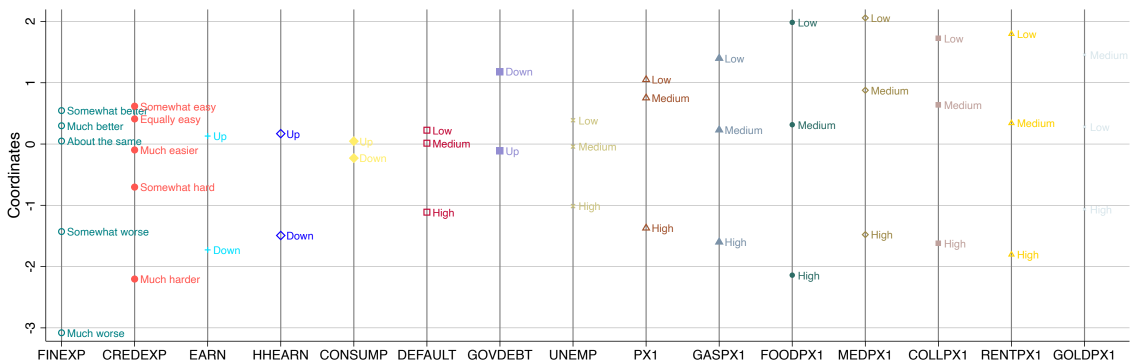

Figure B2: SCE MCA Loadings

Notes: loadings for our baseline MCA estimates using the SCE. Each point on the x-axis corresponds to an included question; the scatter points represent the estimated loadings for each categorical response. Included questions: FINEXP: expectations of personal financial conditions. CREDEXP: expectations regarding ease of credit access. EARN: individual earnings expectations. HHEARN: total household earnings expectations. CONSUMP: total household consumption expectations. DEFAULT: expectations regarding the probability of a personal default. GOVDEBT: expectations regarding government debt. UNEMP: expectations regarding the probability of unemployment increasing. PX1: inflation expectations. GASPX1: gas price expectations. FOODPX1: food price expectations. MEDPX1: medical care price expectations. COLLPX1: college tuition expectations. RENTPX1: rental price expectations. GOLDPX1: gold price expectations. Unemployment and price expectations are binned into terciles.

Table B2: SCE MCA Summary

Panel A:		Baseline	
	(1)	(2)	(3)
Dim 1 %	70.3	53.5	51.6
Dim 2 %	9.2	8.9	8.8
Base Corr.		0.980	0.974
Obs.	87,486	87,444	87,078
Start Date	2013	2013	2013
Panel B:		Education	
	(1)	(2)	(3)
Dim 1 %	71.5	71.6	68.6
Dim 2 %	10.1	8.7	9.7
Base Corr.	0.997	0.999	0.999
Obs.	7,987	25,529	53,819
Start Date	2013	2013	2013
Panel C:		Income	
	(1)	(2)	(3)
Dim 1 %	71.3	70.6	67.1
Dim 2 %	9.6	9.0	9.5
Base Corr.	0.999	0.999	0.999
Obs.	23,216	32,650	30,960
Start Date	2013	2013	2013
Panel D:		Age	
	(1)	(2)	(3)
Dim 1 %	68.6	70.1	69.1
Dim 2 %	11.5	8.9	8.2
Base Corr.	0.999	0.999	0.999
Obs.	32,034	41,420	14,006
Start Date	2013	2013	2013

Notes: MCA results for the SCE. Panel A estimates various MCAs across all consumers, while Panels B, C, and D restrict the sample to different subgroups (as described in Table B1). The first column of Panel A estimates our baseline SCE MCA; included questions are described in Figure B2. Column (2) adds additional backward-looking questions regarding personal financial conditions and credit access conditions. Column (3) adds additional questions regarding future stock price expectations and interest rate expectations.

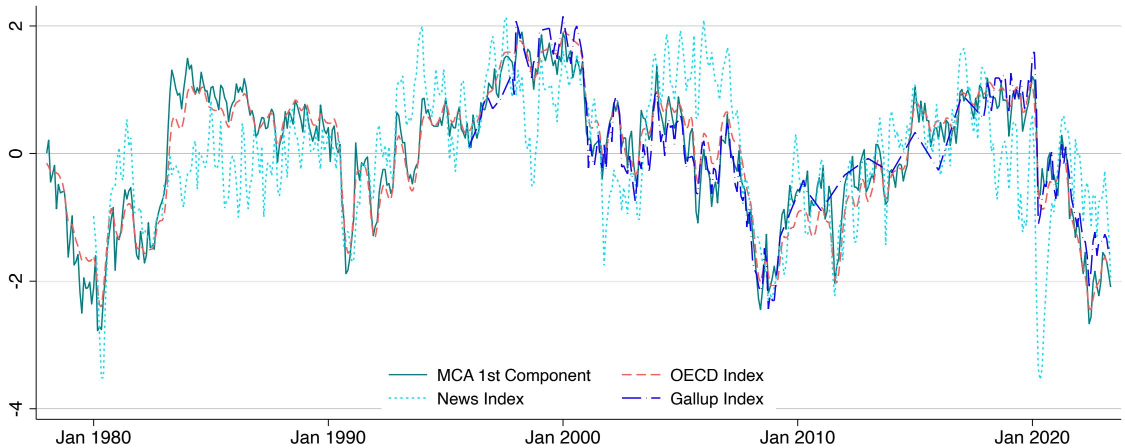

Figure B3: Comparison of MCA Component and Popular Indices of Sentiment

Notes: time-series comparison of different normalized measures of U.S. consumer sentiment and the average value of the fitted first component from our MCA analysis. The solid line represents the average value of our fitted first component (averaged over consumers in each month). The dashed line is the "Consumer confidence index" for the U.S. from the OECD. The dashed-dotted line is Gallup's "Economic Confidence Index." The dotted line is a "News Sentiment index" from Shapiro et al. (2022).

Table B3: SPF PCA Summary

	Dim 1	Dim 2	Dim 3	Dim 4
Nominal Growth (Next Quarter)	0.377	-0.011	-0.130	0.104
Nominal Growth (Next Year)	0.317	0.139	-0.111	0.275
Inflation (Next Quarter)	0.055	0.071	0.656	0.053
Inflation (Next Year)	0.066	0.074	0.658	0.051
Corporate Profit Growth (Next Quarter)	0.274	0.009	0.128	0.320
Corporate Profit Growth (Next Year)	0.236	0.178	0.084	0.418
Unemployment Change (Next Quarter)	-0.354	0.096	0.103	0.215
Unemployment Change (Next Year)	-0.378	0.016	0.081	0.170
Industrial Production Growth (Next Quarter)	0.386	-0.136	-0.051	-0.055
Industrial Production Growth (Next Year)	0.365	0.030	-0.052	0.070
Housing Starts Growth (Next Quarter)	0.176	0.424	0.048	-0.474
Housing Starts Growth (Next Year)	0.068	0.525	0.048	-0.413
T-Bill Rate Change (Next Quarter)	0.120	-0.513	0.164	-0.274
T-Bill Rate Change (Next Year)	0.137	-0.437	0.172	-0.279
% Explained	35.116	14.714	14.112	10.711

Notes: SPF PCA estimates. The tables report the estimated loading for each forecast variable across the first four dimensions. The bottom row explains the fraction explained for the first four dimensions.

Table B4: SPF Psuedo MCA Summary

Panel A:	Dimension 1				
	(1)	(2)	(3)	(4)	(5)
Nominal Growth (Next Quarter)	1.89	0.75	-0.01	-0.51	-1.89
Nominal Growth (Next Year)	1.71	0.88	-0.07	-0.54	-1.81
Inflation (Next Quarter)	0.40	0.19	-0.05	-0.11	-0.50
Inflation (Next Year)	0.67	0.06	0.00	-0.28	-0.62
Corporate Profit Growth (Next Quarter)	1.75	0.59	-0.03	-0.75	-1.81
Corporate Profit Growth (Next Year)	1.66	0.52	-0.06	-0.74	-1.46
Unemployment Change (Next Quarter)	-1.93	-0.51	0.40	0.82	1.95
Unemployment Change (Next Year)	-2.02	-0.58	0.26	0.89	1.96
Industrial Production Growth (Next Quarter)	1.99	0.65	0.05	-0.75	-2.15
Industrial Production Growth (Next Year)	1.93	0.77	0.06	-0.82	-2.20
Housing Starts Growth (Next Quarter)	0.60	0.34	-0.21	-0.21	-0.62
Housing Starts Growth (Next Year)	0.29	0.16	-0.22	-0.19	0.00
T-Bill Rate Change (Next Quarter)	0.96	0.12	0.17	-0.05	-1.02
T-Bill Rate Change (Next Year)	1.17	0.35	0.06	-0.21	-1.22
Panel B:	Dimension 2				
	(1)	(2)	(3)	(4)	(5)
Nominal Growth (Next Quarter)	2.06	-0.71	-1.17	-0.84	1.00
Nominal Growth (Next Year)	2.37	-0.61	-0.97	-0.88	0.60
Inflation (Next Quarter)	0.69	-0.40	-0.45	-0.78	1.02
Inflation (Next Year)	0.84	-0.43	-0.70	-0.80	1.13
Corporate Profit Growth (Next Quarter)	1.12	-0.35	-0.98	-0.77	1.29
Corporate Profit Growth (Next Year)	1.50	-0.17	-0.73	-0.77	0.34
Unemployment Change (Next Quarter)	1.97	-0.88	-1.00	-0.85	1.54
Unemployment Change (Next Year)	1.07	-0.83	-1.07	-0.64	1.52
Industrial Production Growth (Next Quarter)	1.42	-0.91	-1.28	-0.93	1.76
Industrial Production Growth (Next Year)	1.62	-0.55	-1.20	-1.02	1.39
Housing Starts Growth (Next Quarter)	0.93	-0.45	-0.68	-0.40	0.85
Housing Starts Growth (Next Year)	1.19	-0.28	-0.33	-0.50	0.03
T-Bill Rate Change (Next Quarter)	-0.71	-0.98	-0.35	0.01	1.73
T-Bill Rate Change (Next Year)	-0.30	-0.74	-0.27	-0.40	1.52
Panel C:	Fraction Explained				
	(1)	(2)	(3)	(4)	(5)
% Explained:	38.8	19.7	6.9	3.7	2.3

Notes: SPF "pseudo-MCA" estimates. We first convert the continuous responses in the SPF into quintiles. We then estimate an MCA using these categorical responses. Panel A reports the loadings of the first component for each question category, while Panel B reports the loadings of the second component. Panel C reports the fraction explained for the first five dimensions.

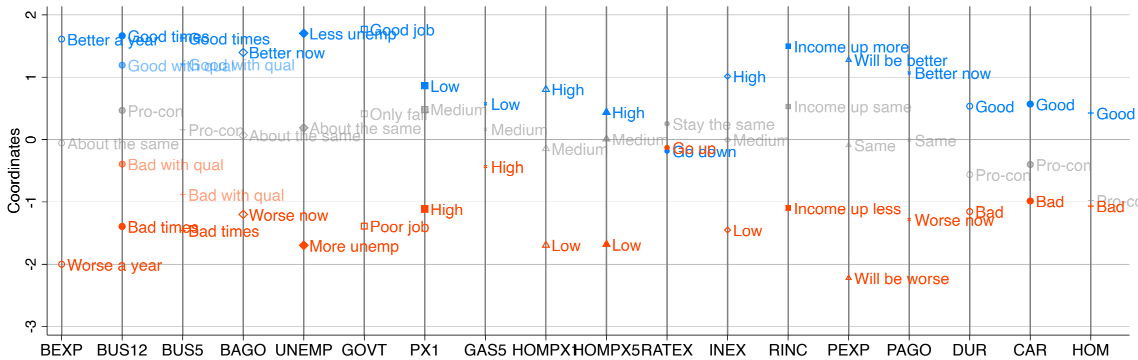

Figure B4: MSC MCA, Gas and Home Prices

Notes: coordinates of an alternative MCA estimated using the MSC. We additionally include questions regarding home price expectations (HOMPX1 and HOMPX5). These questions were introduced in 2007 and are only asked to consumers who own a home.

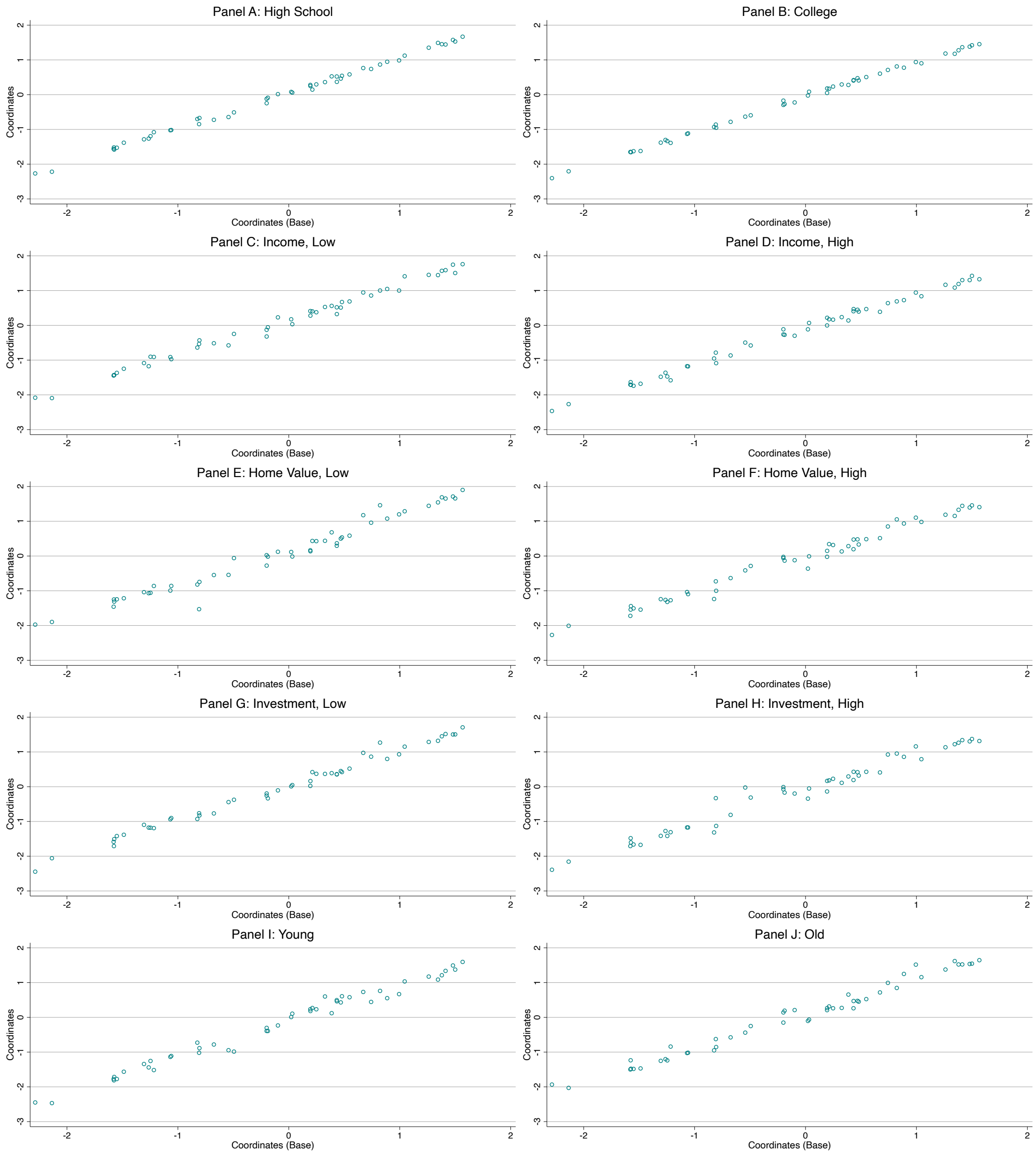

Figure B5: MSC MCA Coordinates Across Demographics

Notes: scatter plots of the estimated MCA loadings across different demographic groups, compared with the baseline pooled across all consumers (from Table 3).

Figure B6: MSC MCA, Rolling

Notes: results from a rolling MCA analysis using the MSC data. We estimate our baseline MCA, rolling over one-year windows. Panel A reports the fraction explained from the first two components. Panel B reports the correlation of the fitted first component with our baseline fitted first component (from Table 3).

Appendix C Model Appendix

C.1 Steady State and Log-Linearization Details

In this appendix, we provide additional details on the steady state and log-linearization of the model.

The central bank chooses the long-run policy rate $i^* \equiv -\log\beta$ which implies that steady-state inflation $\bar{\Pi} = 1 \iff \bar{\pi} = 0$. This implies that long-run real rates $\bar{r} = -\log\beta$, and hence steady-state holdings of (real) bonds $\bar{B} = 0$. The optimal production subsidy implies that in steady-state, real wages satisfy

$$\bar{W} = (1+\tau^S)\left(\frac{\epsilon-1}{\epsilon}\right) \equiv 1$$

Additionally, in steady state every firm chooses the same price, thus there is no price dispersion, and so

$$\bar{Y} = \bar{C} = \bar{N}$$

This implies that steady state profits are zero, as are transfers:

$$\bar{D} = (1-\tau^S)\bar{Y} - \bar{W}\bar{N} - \tau^S\bar{Y} = 0$$

Since steady-state bond holdings are also zero, household transfers $\bar{T}^H = \bar{T}^F = 0$. Combining the intratemporal optimality conditions with the budget constraints at steady state (and normalizing the steady-state labor disutility shock $\bar{\Gamma} = 1$) gives

$$\begin{gathered} \bar{C}^S = \bar{W}\bar{N}^S, \;\; \bar{C}^H = \bar{W}\bar{N}^H \\ \bar{W} = \bar{\Gamma}(\bar{C}^S)^\sigma(\bar{N}^S)^\phi, \;\; \bar{W} = \bar{\Gamma}(\bar{C}^H)^\sigma(\bar{N}^H)^\phi \\ \implies \bar{C} = \bar{C}^S = \bar{C}^H, \;\; \bar{N} = \bar{N}^S = \bar{N}^H \end{gathered}$$

Then since $\bar{C}^H = \bar{C}^S$ and $\bar{N}^H = \bar{N}^S$, we have

$$y_t = c_t = \lambda c_t^H + (1-\lambda)c_t^S \tag{C1}$$

$$y_t = n_t = \lambda n_t^H + (1-\lambda)n_t^S \tag{C2}$$

since price dispersion is zero to a first order (see Galí 2015). Profits are given by

$$d_t = -w_t$$

Thus we have that the S optimality conditions are given by equations (9). The log-linearized firm optimality conditions for optimal update price P_t^* imply (see Galí 2015):

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)}{\theta} w_t + \beta \mathbb{E}_t \pi_{t+1}$$

Finally, the linearized H budget constraint is given by $c_t^H = n_t^H + \left(1 - \tau^D/\lambda\right) w_t$, which combined with the intratemporal optimality conditions gives equations (11).

C.2 Kalman Prior and Posterior Invertibility

In this appendix, we show that whenever $\mu > 0$, the time-invariant prior and posterior covariance matrices are invertible.

Note that have

$$\begin{aligned}
\mathbf{\Sigma}_{1|1} &= (\mathbf{I} - \mathbf{KH})\, \mathbf{\Sigma}_{1|0} \\
&= \mathbf{\Sigma}_{1|0} - \mathbf{\Sigma}_{1|0}^{1/2} \mathbf{U}_1 \left(\mathbf{I}_1 + \mathbf{\Sigma}_\eta\right)^{-1} \mathbf{U}_1^\top \mathbf{\Sigma}_{1|0}^{1/2} \\
&= \mathbf{\Sigma}_{1|0}^{1/2} \mathbf{U} \begin{bmatrix} \frac{\mu}{2} \cdot \mathbf{\Lambda}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix} \mathbf{U}^\top \mathbf{\Sigma}_{1|0}^{1/2}
\end{aligned}$$

where the final line follows from

$$1 + \sigma_{\eta,i} = 1 + \frac{1}{2\Lambda_i/\mu - 1} \implies \left(\mathbf{I}_1 + \mathbf{\Sigma}_\eta\right)^{-1} = \mathbf{I}_1 - \frac{\mu}{2} \cdot \mathbf{\Lambda}_1^{-1}$$

Note that whenever $\mu > 0$, the block diagonal matrix above is invertible. Thus, if $\mathbf{\Sigma}_{1|0}$ is invertible, so is $\mathbf{\Sigma}_{1|1}$.

Additionally, we have

$$\mathbf{\Sigma}_{1|0} = \mathbf{A}\mathbf{\Sigma}_{1|1}\mathbf{A}^\top + \mathbf{C}\mathbf{C}^\top = \begin{bmatrix} \begin{bmatrix}\mathbf{A}_x & \mathbf{C}_x\end{bmatrix} \mathbf{\Sigma}_{1|1} \begin{bmatrix}\mathbf{A}_x^\top \\ \mathbf{C}_x^\top\end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Then if $\mathbf{\Sigma}_{1|1}$ is invertible and assuming $\begin{bmatrix}\mathbf{A}_x & \mathbf{C}_x\end{bmatrix}$ is full row rank, the upper-left block above is positive definite (since $\mathbf{\Sigma}_{1|1}$ is positive definite). Thus $\mathbf{\Sigma}_{1|0}$ is invertible.

As $\mu \to 0$, if $\mathbf{\Omega}$ is full rank, then for μ small enough, all eigenvalues will satisfy the conditions in Proposition 1 and so the block diagonal matrix simply becomes $\frac{\mu}{2} \cdot \mathbf{\Lambda}^{-1}$, so as $\mu \to 0$ $\mathbf{\Sigma}_{1|1} \to \mathbf{0}$. However, when $\mathbf{\Omega}$ is not full rank, there are are set of zero eigenvalues, thus as $\mu \to 0$, we have

$$\begin{bmatrix} \frac{\mu}{2} \cdot \mathbf{\Lambda}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix} \to \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix}$$

Thus, $\mathbf{\Sigma}_{1|1}$ converges to a non-zero (singular) matrix.

C.3 Hand-to-Mouth Quadratic Utility

In this appendix, we formally derive the log-quadratic approximation of hand-to-mouth household utility.

The information-constrained households choose labor N_t^j, and consumption C_t^j is determined as a residual. Write the concentrated utility function as

$$\mathcal{U}(N_t^j; W_t, D_t, \Gamma_t) \equiv \frac{\left(W_t N_t^j + (\tau^D/\lambda) D_t\right)^{1-\varsigma} - 1}{1-\varsigma} - \Gamma_t \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi}$$

Re-write all variables in terms of log deviations from the steady state (for any variable $X_t \equiv \bar{X} e^{x_t}$, except aggregate profits, where we instead have $D_t = \bar{Y} d_t$). Then taking derivatives with respect to the choice variables $n_t^{H,j}$ evaluated at the steady state gives

$$\left.\frac{\partial \mathcal{U}}{\partial n_t^{H,j}}\right|_{SS} = 0, \quad \left.\frac{\partial^2 \mathcal{U}}{\partial (n_t^{H,j})^2}\right|_{SS} = -(\varsigma + \varphi)$$
$$\left.\frac{\partial^2 \mathcal{U}}{\partial n_t^{H,j} \partial w_t}\right|_{SS} = 1 - \varsigma, \quad \left.\frac{\partial^2 \mathcal{U}}{\partial n_t^{H,j} \partial \gamma_t}\right|_{SS} = -1, \quad \left.\frac{\partial^2 \mathcal{U}}{\partial n_t^{H,j} \partial d_t}\right|_{SS} = -\varsigma$$

Next, from our log-linearization we have that $d_t = -w_t$. Finally, define the (endogenous) vectors $\mathbf{A}_w$ and $\mathbf{A}_\gamma$ so that $\mathbf{A}_w^\top \mathbf{X}_t = w_t$ and $\mathbf{A}_\gamma^\top \mathbf{X}_t = \gamma_t$ in equilibrium. Then we have that the quadratic approximation for H household utility is given by (18), where

$$\mathbf{B}_{aa} = \frac{1}{2}(\varsigma + \varphi) \tag{C3}$$
$$\mathbf{B}_{ax} = \begin{bmatrix} \mathbf{A}_w & \mathbf{A}_\gamma \end{bmatrix} \begin{bmatrix} \chi_n \\ -1 \end{bmatrix} \tag{C4}$$

Note that $\mathbf{B}_{aa}$ is a scalar, and hence the loss matrix $\mathbf{\Omega}$ from (21) is rank one and the eigenvector associated with the only nonzero eigenvalue is (proportional to) (C4):

$$\mathbf{\Omega} = \mathbf{v}\mathbf{v}^\top, \quad \mathbf{v} \equiv \left(\frac{1}{4}\frac{1}{\sqrt{\varsigma + \varphi}}\right) \cdot \mathbf{\Sigma}_{1|0}^{1/2} \mathbf{B}_{ax}$$
$$\implies \Lambda_1 = \mathbf{v}\mathbf{v}^\top, \quad \mathbf{u}_1 = \Lambda_1^{-1} \cdot \mathbf{v}$$

Then we have that the signal coefficient matrix is a row vector

$$\begin{aligned}\mathbf{H} &= \Lambda_1^{-1} \cdot \mathbf{v} \mathbf{\Sigma}_{1|0}^{-1/2} \\ &= \Lambda_1^{-1} \left(\frac{1}{4}\frac{1}{\sqrt{\varsigma + \varphi}}\right) \cdot \mathbf{B}_{ax}\end{aligned}$$

and the signal noise covariance is a scalar:

$$\mathbf{\Sigma}_\eta \equiv \sigma_\eta^2 = (2\Lambda_1/\mu - 1)^{-1}$$

assuming that $\Lambda_1 > \frac{1}{2}\mu$. Define the following transformed (scalar) Kalman gain

$$K \equiv \mathbf{H}\mathbf{K} = \frac{1}{1 + \sigma_\eta^2}$$

which follows from the general result above regarding **HK**.

Note from the definition of $\mathbf{B}_{ax}$ in this case, we find

$$\mathbf{B}_{ax}^\top \mathbf{X}_t = \begin{bmatrix} \chi_n & -1 \end{bmatrix} \begin{bmatrix} w_t \\ \gamma_t \end{bmatrix} = \chi_n w_t - \gamma_t \equiv (\varsigma + \varphi) n_t^{H,*}$$

$$\implies \mathbf{H}\mathbf{X}_t = \Lambda_1^{-1} \left(\frac{1}{4} \frac{1}{\sqrt{\varsigma + \varphi}} \right) \cdot (\varsigma + \varphi) n_t^{H,*}$$

$$\mathbf{H}\hat{\mathbf{X}}_t^j = \Lambda_1^{-1} \left(\frac{1}{4} \frac{1}{\sqrt{\varsigma + \varphi}} \right) \cdot (\varsigma + \varphi) \hat{n}_t^{H,*,j}$$

$$\mathbf{H}\tilde{\mathbf{X}}_t^j = \Lambda_1^{-1} \left(\frac{1}{4} \frac{1}{\sqrt{\varsigma + \varphi}} \right) \cdot (\varsigma + \varphi) \tilde{n}_t^{H,*,j}$$

Then we have

$$\hat{\mathbf{X}}_t^j = (\mathbf{I} - \mathbf{K}\mathbf{H}) \tilde{\mathbf{X}}_t^j + \mathbf{K} s_t^j$$

$$\implies \hat{n}_t^{H,*,j} = K \left(n_t^{H,*} + \eta_t^j \right) + (1 - K) \tilde{n}_t^{H,*,j}$$

C.4 Equilibrium Response Coefficients (No Dynamics)

The coefficients in equations (25) are given by

$$C_{y,v} \equiv \frac{1 - \lambda}{(1-\lambda)\tilde{\omega}_y \kappa_w \phi_\pi + \varsigma(1 - \lambda\tilde{\zeta}_y)} \tag{C5}$$

$$C_{y,\gamma} \equiv \frac{\varsigma\lambda\tilde{\zeta}_\gamma - (1-\lambda)\tilde{\omega}_\gamma \kappa_w \phi_\pi}{(1-\lambda)\tilde{\omega}_y \kappa_w \phi_\pi + \varsigma(1 - \lambda\tilde{\zeta}_y)} \tag{C6}$$

$$C_{\pi,v} \equiv \frac{(1-\lambda)\kappa_w \tilde{\omega}_y}{(1-\lambda)\tilde{\omega}_y \kappa_w \phi_\pi + \varsigma(1 - \lambda\tilde{\zeta}_y)} \tag{C7}$$

$$C_{\pi,\gamma} \equiv \frac{\varsigma\kappa_w \left(\tilde{\omega}_\gamma (1 - \lambda\tilde{\zeta}_y) + \lambda\tilde{\omega}_y \tilde{\zeta}_\gamma \right)}{(1-\lambda)\tilde{\omega}_y \kappa_w \phi_\pi + \varsigma(1 - \lambda\tilde{\zeta}_y)} \tag{C8}$$

where

$$\tilde{\omega}_\gamma \equiv \frac{1-\lambda(1-K)}{1-\chi_n\lambda(1-K)} \tag{C9}$$

$$\tilde{\omega}_y \equiv \frac{\varsigma+\varphi}{1-\chi_n\lambda(1-K)} \tag{C10}$$

$$\tilde{\zeta}_\gamma \equiv \frac{\varsigma^{-1}(1-\chi_n)(\varphi(1-\lambda(1-K))+\varsigma(1-K)(1-\lambda))}{(\varsigma+\varphi)(1-\chi_n\lambda(1-K))} \tag{C11}$$

$$\tilde{\zeta}_y \equiv \frac{\varsigma^{-1}(1-\chi_n)\varphi+1-\chi_n(1-K)}{1-\chi_n\lambda(1-K)} \tag{C12}$$

C.4.1 Expectations Manipulation (No Dynamics)

Following the same steps as in Section 3, we find

$$\begin{aligned}
w_t &= \tilde{\omega}_\gamma\gamma_t + \tilde{\omega}_y y_t + \tilde{\omega}_z z_t \\
c_t^H &= \tilde{\zeta}_\gamma\gamma_t + \tilde{\zeta}_y y_t + \tilde{\zeta}_z z_t \\
\implies y_t &= (1-\lambda\tilde{\zeta}_y)^{-1}\left[(1-\lambda)\varsigma^{-1}(v_t-\phi_\pi\pi_t)+\lambda\tilde{\zeta}_\gamma\gamma_t+\lambda\tilde{\zeta}_z z_t\right] \\
\pi_t &= \kappa_w\tilde{\omega}_\gamma\gamma_t + \kappa_w\tilde{\omega}_y y_t
\end{aligned}$$

where we additionally have the terms related to the expectation shock:

$$\begin{aligned}
\tilde{\omega}_z &= -K\frac{\lambda(\varsigma+\varphi)}{1-\lambda\chi_n(1-K)} \\
\tilde{\zeta}_z &= -K\frac{\varsigma^{-1}(1-\chi_n)\lambda\varphi-(1-\lambda)}{1-\lambda\chi_n(1-K)}
\end{aligned}$$

Appendix D Additional Model Output

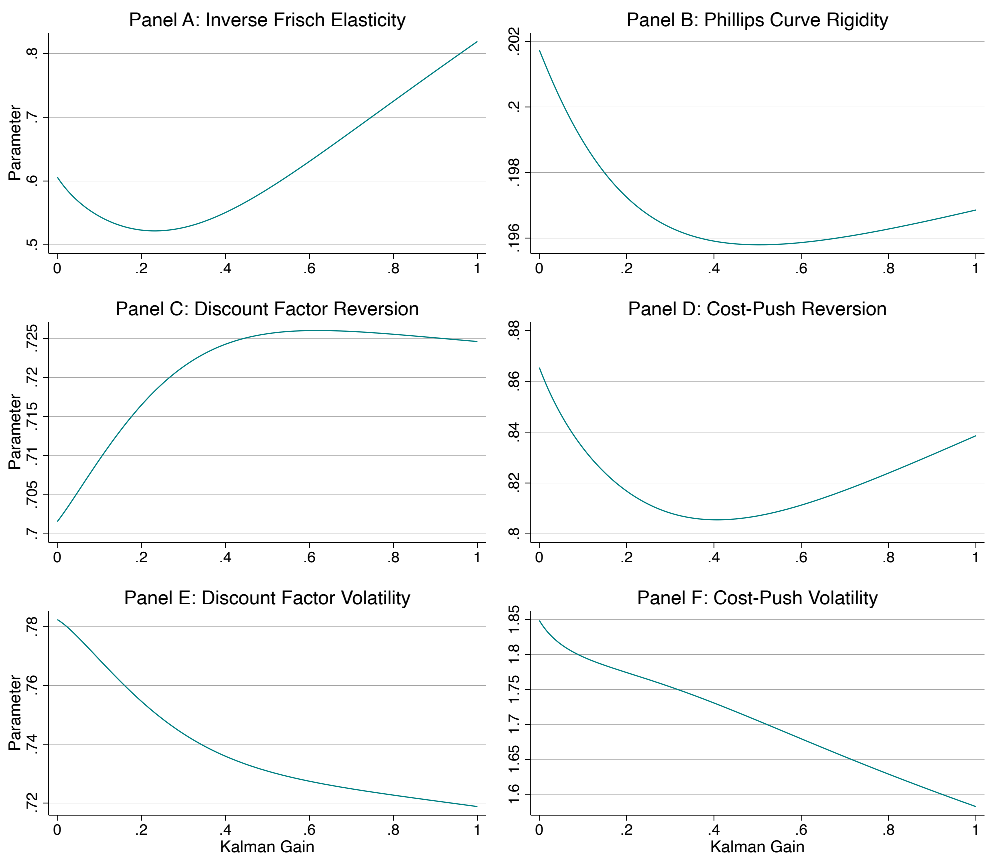

Figure D1: Estimated Parameters: Varying Information Frictions

Notes: parameter estimates of the model as we vary the Kalman gain parameter $K \in (0, 1)$. For each point on the x-axis, we re-estimate the model targeting the same set of moments in Table 4 (besides the correlation of inflation beliefs and inflation). Each panel corresponds to the different parameters we calibrate.

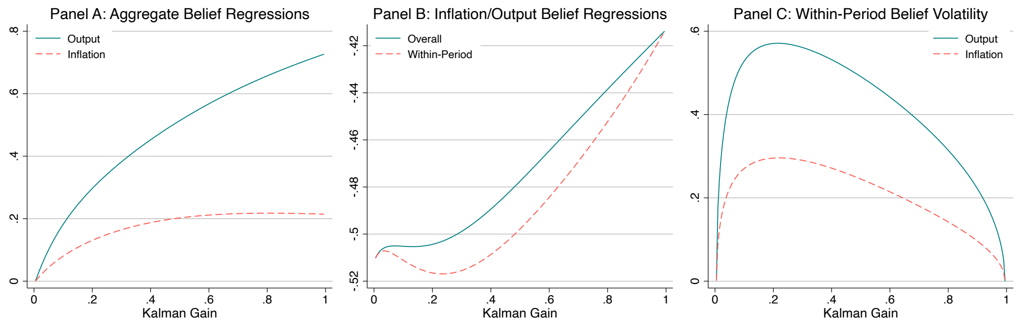

Figure D2: Beliefs as a Function of Information Costs

Notes: model-implied moments as a function of information costs K. Panel A reports model-implied regression coefficients of $\hat{y}_t^j$ on y_t (solid line) or $\hat{\pi}_t^j$ on π_t (dashed line). Panel B reports model-implied regression coefficients of $\hat{\pi}_t^j$ on $\hat{y}_t^j$; the solid line reports unconditional coefficients, while the dashed line is cross-sectional (across $j \in [0, \lambda]$). Panel C reports the cross-sectional volatility of beliefs regarding $\hat{y}_t^j$ (solid line) and $\hat{\pi}_t^j$ (dashed line).

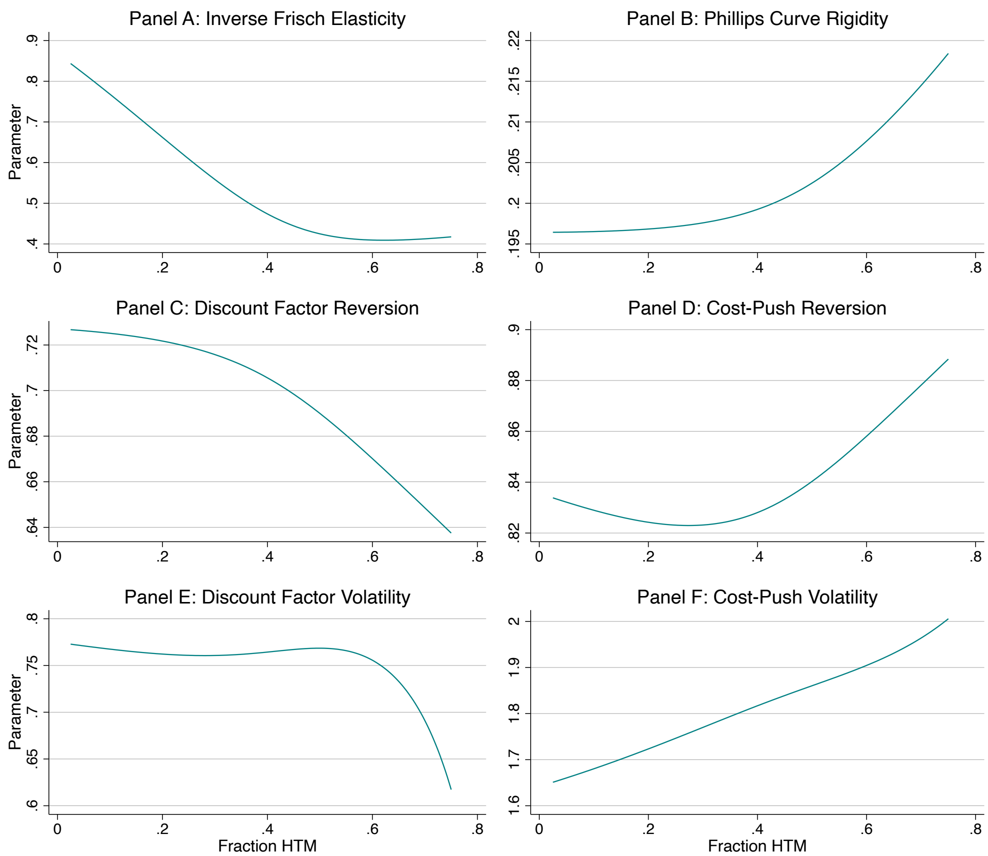

Figure D3: Estimated Parameters: Varying Hand-to-Mouth Fraction

Notes: parameter estimates of the model as we vary the fraction of H households $\lambda \in (0, 1)$. For each point on the x-axis, we re-estimate the model targeting the same set of moments in Table 4. Each panel corresponds to the different parameters we calibrate.

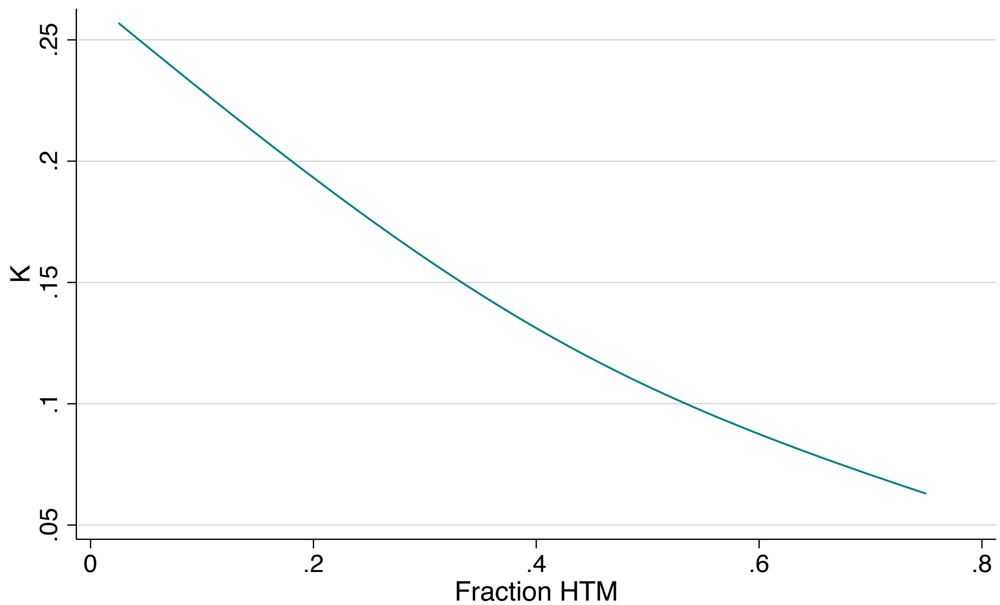

Figure D4: Parameters Information Frictions: Varying Hand-to-Mouth Fraction

Notes: Kalman gain K estimates of the model as we vary the fraction of H households $\lambda \in (0, 1)$. For each point on the x-axis, we re-estimate the model targeting the same set of moments in Table 4.

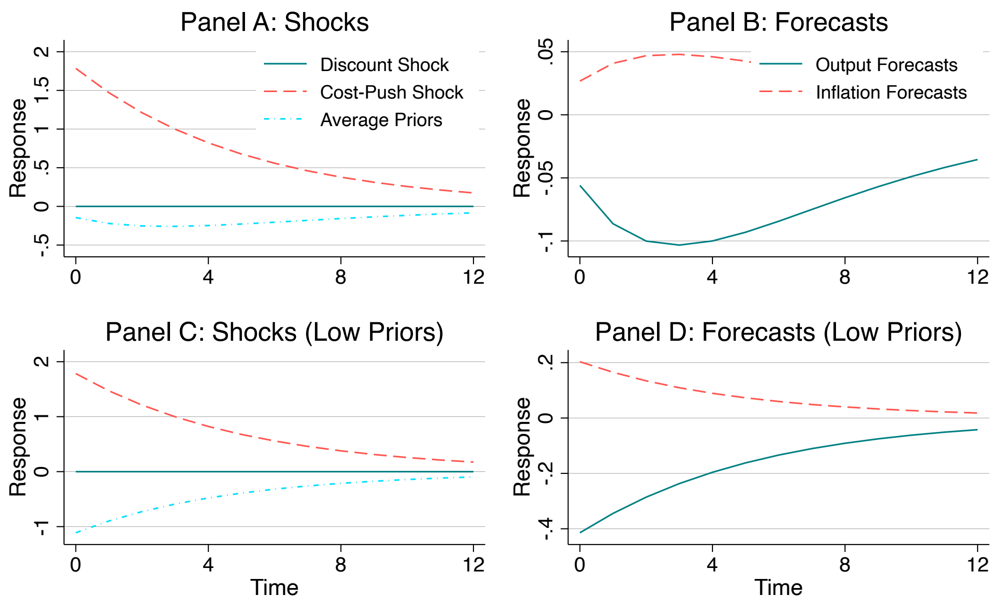

Figure D5: Response to Supply Shock: State Variables and 1-Year Ahead Forecasts

Notes: IRFs of the model state variables v_t, γ_t, m_t and the 1-year ahead output and inflation posterior beliefs $\hat{y}^j_{t+4}, \hat{\pi}^j_{t+4}$ following the supply shock considered in Figure 3.

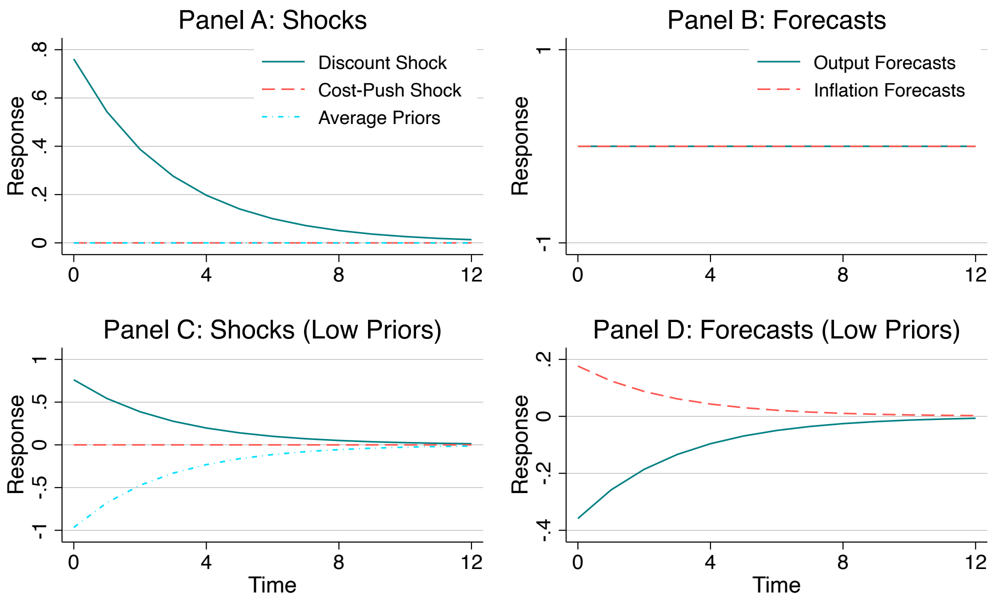

Figure D6: Response to Demand Shock: State Variables

Notes: IRFs of the model state variables v_t, γ_t, m_t and the 1-year ahead output and inflation posterior beliefs $\hat{y}^j_{t+4}, \hat{\pi}^j_{t+4}$ following the demand shock considered in Figure 4.